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On Skolem's theorem.

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In 1922, Th. Skolem proved the following famous theorem: If there exists a model of any cardinal number for a system of axioms (satisfying certain conditions), then there exists also a countable model for the system. The aim of the present paper is to formulate and prove a corresponding theorem from the finite stand point. Our theorem reads as follows:

MAIN THEOREM. If A, B, C, D, E in Gödel [2] are consistent, then A, B, C, D, E and the following axioms are consistent.

$$\forall x \exists y (y \in \omega \wedge f_0(y) = x)$$

$$\forall x \forall y (x = y \vdash f_0(x) = f_0(y)),$$

where f_0 is a function, which is not contained in axioms A—E, and ω has the same meaning as in Gödel [2].

Our proof depends on results of our former paper [6], [7] which, in turn, is based on [8]. In [8], we have generalized LK (Logistischer klassischer Kalkül) of Gentzen [4] to GLC (Generalized logic calculus). Especially we shall make use of the "restriction theory" (§ 7) of [8]. In [6] we have treated in detail G^1LC, a specialization of GLC, and established the theorem: The fundamental conjecture holds for normal proof-figure. (Both terms: "fundamental conjecture" and "normal proof-figure" are defined in [6].) We shall now call $\tilde{L}K$, a logical system obtained from G^1LC by restricting it as follows:

In every $\forall$ left on f-variable of the form

$$\frac{F(H), \Gamma \rightarrow \Delta}{\forall \varphi F(\varphi), \Gamma \rightarrow \Delta}$$

$F(\alpha)$ is not allowed to have any $\forall$ on f-variable. And the beginning sequence of $\tilde{L}K$ is not allowed to have any logical symbol. We see that every proof-figure of $\tilde{L}K$ is normal (in the sense defined in [6]).

Now we introduce two definitions in the system LK of Gentzen.