Journal of the Mathematical Society of Japan Vol. 6, Nos. 3~4, December, 1954

On the radial order of a certain regular function in a unit circle.

By Masatsugu TSUJI

(Received April 29, 1954)

1. Seidel and Walsh[1] proved the following theorem.

THEOREM 1. *Let* $w=f(z)$ *be regular and univalent in* $|z|<1$. *Then there exists a null set* E *on* $|z|=1$, *such that if* $e^{i\theta}$ *does not belong to* E, *then*

$$f'(z)=o\left(\frac{1}{\sqrt{|z-e^{i\theta}|}}\right) \qquad \text{uniformly for a fixed } \theta,$$

when $z\rightarrow e^{i\theta}$ *from the inside of any Stolz domain, whose vertex is at* $e^{i\theta}$.

We shall give a simple proof as follows.

PROOF. Let D be the image of $|z|<1$ on the w-plane. Then since by an elementary transformation, we can map D on a finite domain, we may assume that D is a finite domain, so that

$$\int_0^{2\pi}\int_0^1 |f'(re^{i\theta})|^2 r dr d\theta < \infty ,$$

hence for almost all θ in $[0, 2\pi]$,

$$\int_0^1 |f'(re^{i\theta})|^2 dr < \infty . \tag{1}$$

Let (1) hold for $\theta=0$ and we shall prove that

$$f'(z)=o\left(\frac{1}{\sqrt{|z-1|}}\right) \quad \text{uniformly,} \tag{2}$$

1) W. Seidel and J. L. Walsh: On the derivatives of functions analytic in the unit circle and their raddii of univalence and of p-valence. Trans. Amer. Math. Soc. 52 (1942). F. Ferrand: C. R. Acad. des Sci. du 10 novembre 1941 and Thèse du 12 janvier 1942. J. Wolf: Inégalités remplies par derivées des fonctions holomorphes, univalentes et bornées dans un demi-plan. Commentarii Math. Helvetici. 45 (1952-53).