Journal of the Mathematical Society of Japan Vol. 5. Nos. 3~4. November, 1953.

Some problems of minima concerning the oval.[*]

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§ 1. As is well known, the maximum area of those ovals which carry in every direction an assigned breadth has been found by use of the method of central symmetrization; this method transforms any oval into a central oval having greater area and the same breadth with the given oval in every direction. But, the method to make the area smaller is not yet known. We think this is the main difficulty in minimum problems of ovals under some conditions on the breadth.

However, some special cases in this direction were studied by Hayashi, Pál and others, as an analogue to the isoperimetric problem or as a solution of Kakeya's problem. The main inequalities which have been got are the following:

(1) $F \geqq \varDelta^2/\sqrt{3}$ (Pál)[1];

(2) $2F > \varDelta D$ (Kubota)[2];

(3) $4F \geqq \varDelta L - b\varDelta$ when $2\sqrt{3}\,\varDelta \leqq L$,

where b is a positive root smaller than $2\varDelta/\sqrt{3}$ of the equation

$2Lx^3-(L^2-\varDelta^2)x^2-2L\varDelta^2x+L^2\varDelta^2=0$ (Yamanouchi)[3];

(4) $4F \geqq (L-2D)\sqrt{4LD-L^2}$, when $2D < L \leqq 3D$ (Kubota)[2];

(5) $2F \geqq (\pi-\sqrt{3})B^2$, when $B=D=\varDelta=L/\pi$ (Lebesgue[4][5] and Blaschke)[6];

(6) $4F \geqq (L-2D)\sqrt{3}D$, when $3D \leqq L \leqq \pi D$ (Kubota)[7].

In these inequalities we denote by F the area, by L the perimeter, by D the diameter which is the length of the greatest chord and at the same time the length of the greatest breadth, and by $\varDelta$ the length of the smallest breadth. The minimum ovals in the cases (1)~(5) are respectively the following:

(1) a regular triangle whose height is $\varDelta$ when $\varDelta$ is given;