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On the capacity of general Cantor sets.

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1. General linear Cantor sets.

1. Let Δ be an interval on the x-axis. We take $k\,(\geqq 2)$ disjoint intervals $\Delta_{i_1}\,(i_1=1,2,\cdots,k)$ in Δ and k disjoint intervals $\Delta_{i_1 i_2}\,(i_2=1,2,\cdots,k)$ in Δ_{i_1} and proceed similarly, then after n steps, we obtain k^n intervals $\Delta_{i_1\cdots i_n}\,(i_1,\cdots,i_n=1,2,\cdots,k)$, such that

$$\Delta_{i_1\cdots i_n}\subset\Delta_{i_1\cdots i_{n-1}}.\quad (i_n=1,2,\cdots,k). \tag{1}$$

We put

$$E=\prod_{n=1}^{\infty}\Big(\sum_{i_1,\cdots,i_n}^{1,2,\cdots,k}\Delta_{i_1\cdots i_n}\Big). \tag{2}$$

In §1 and §2, we denote the length of an interval I by $|I|$ and the logarithmic capacity of a set M by $\gamma(M)$. We assume that there exists constants $a>0$, $b>0$, such that for $n=1,2,\cdots$

$$|\Delta_{i_1\cdots i_{n-1}\nu}|\geqq a\,|\Delta_{i_1\cdots i_{n-1}}|\quad (\nu=1,2,\cdots,k) \tag{3_1}$$

and

the mutual distance of $\Delta_{i_1\cdots i_{n-1}\mu}$ and $\Delta_{i_1\cdots i_1\cdots i_{n-1}\nu}\,(\mu,\nu=1,2,\cdots,k,\mu\neq\nu)$ is $\geqq b\,|\Delta_{i_1\cdots i_{n-1}}|$. $\quad(3_2)$

Then we call E a general linear Cantor set.

THEOREM 1. *Let E be a general linear Cantor set. Then*

$$m(E)=0,\qquad \gamma(E)\geqq a^{\frac{1}{k-1}}\,b\,|\Delta|>0,$$

where $m(E)$ is the linear measure and $\gamma(E)$ the logarithmic capacity of E.

At every point of E, the upper capacity density of E is positive, so that every point of E is a regular point for Dirichlet problem.