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# Wedderburn's theorem, weakly normal rings, and the semigroup of ring-classes.

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A well-known theorem of Wedderburn asserts that if a central simple finite-dimensional algebra $A$, over a field, is a subalgebra of an algebra $S$ and if the unit element of $A$ is also a unit element in $S$, then $S$ is the Kronecker product $A \times V_S(A)$, where $V_S(A)$ denotes the commuter ring of $A$ in $S$. An interesting generalization of the theorem was recently obtained by Azumaya [1]. It deals with the notion of maximally central algebras, which was introduced formerly by Azumaya and the writer [2] in a narrower sense, in a different context. In the present note we first offer (§1, Theorem 1 and §3, Theorems, 2, 3) a further generalization of that Wedderburn-Azumaya theorem, dealing simply with a ring $A$ possessing an independent finite right-basis over its (not necessarily commutative) subring $C$. On the other hand, weakly normal (or "galoisien") subrings of a ring have recently been used effectively by Dieudonné [3] and the writer [7], [9], [10] in studying automorphisms and the Galois theory or rings. The innerly weakly normal case is of particular interest in our context, and our theorem can, togetheor with some other propositions, be given a finer formulation in this case (§3). The maximally central case is a further particular case in which the innerly weakly normal subring $C$ is commutative and is contained in (in fact, coincides with) the center of $A$. For maximally central rings Azumaya defined the notion of algebra- or ring-classes and introduced their group, a generalization of the Brauer group of the classes of central simple algebras. We are led to introduce the semigroup of the ring-classes of rings containing a fixed commutative ring $C$ in their center and weakly normal over $C$ (as we want to call) (5). It turns out that Azumaya's group is in fact the largest subgroup in this semigroup (Theorem 5). In Appendix we give a simple proof to Jacobson's inverse to Wedderburn's theorem.