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Symbolic methods in the problem of three-line Latin rectangles.

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1. Introduction. In previous papers [1], [2] the author gave an asymptotic series for the number $f(3, n)$ of $3\times n$ Latin rectangles:

$$f(3, n)\sim e^{-3}(n!)^{3}\sum_{i=0}^{n}H_{i}\left(-\frac{1}{2}\right)/i!(n)_{i}$$

where $H_{i}(x)$ is Hermite polynomial and $(n)_{i}=n!/(n-i)!$ is Jordan factorial. The method of [2] was rather complicated, and the author has attempted in [1] (written actually after [2]) to clarify derivation of the asymptotic series. The present paper will make a slight improvement of [1].

One of the most powerful instruments in such a problem would be the *symbolic method*, recently recognized in its full power by Touchard, Fréchet, Kaplansky and others. This method also clarifies results on the number $f(3, n)$, and our Theorem 1 states $f(3, n)$ as a polynomial in the shift operator (to the left) applying on partial sums for e^{-3}. This explicit form, though complicated in itself, is undoubtedly the most reasonable for the asymptotic expansion (Theorem 2).

Mr. John Riordan of Bell Telephone Laboratories kindly communicated to the author an application of [1], obtaining a very neat recursive formula for the number $f(3, n)$ (*Memorandum*, Sept. 8, 1950). This would be the simplest recursion one can expect, and it is a great pleasure for the author to include it in this paper (Theorem 3).

Our method strongly suggests an extension to the general Latin rectangles. The first and the chief obstacle, however, lies in establishing analogue of Theorem 1, and this would have to be overcome only through elaborate inductions.

Several symbolisms are used in this paper to simplify description. One is the symbolic representation of sieve process (or Poincaré's