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On the extension property of normed spaces over fields with non-archimedean valuations.

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If we wish to develop a theory of normed spaces over non-archimedean fields after the model of the usual theory in archimedean cases, the first thing to do would be to establish an analogue of the Hahn-Banach theorem on linear functionals. We shall examine in the present note in which case this is possible. We shall prove a simple theorem, which answers completely the question when the ground field is e. g. the p-adic field. The idea of the "binary intersection property" given by L. Nachbin in his paper[1] was very useful to our purpose.

Let k be a complete field with a non-trivial discrete valuation $|\ |$. This field k will be fixed throughout this paper.

Suppose a vector space S over k is *normed*[2]; i. e. to each element $x \in S$ corresponds a real number $\|x\|$, which has the properties:

1. $\|x\| \geqq 0$; $\|x\|=0$ *if and only if* $x=0$
2. $\|x+y\| \leqq \|x\| + \|y\|$
3. $\|\alpha x\| = |\alpha|\, \|x\|$ *for all* $\alpha \in k$.

A space S in which the stronger form of the triangular inequality

2'. $\|x+y\| \leqq \mathrm{Max}(\|x\|, \|y\|)$

holds is called *non-archimedean*[3].

A linear and continuous mapping f from a normed space S into k is called a *linear functional*. The set of all such functionals is written by S^*.

As in the ordinary case, we call a linear mapping f from S into k bounded if there exists a real number c such that $|f(x)| \leqq c\|x\|$ for all $x \in S$.