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On Integral Invariants and Betti Numbers of Symmetric Riemannian Manifolds, I.

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In his classical paper E. Cartan proved the important theorem that the *p-th Betti-number of a compact symmetric Riemannian manifold M is equal to the number of linearly independent invariant differentials of rank p defined on M.* Now there exist two kinds of compact symmetric Riemannian manifolds. *The first class* consists of the *group-manifolds of simple compact Lie groups.* The fundamental groups of these manifolds are semi-simple. The Poincaré-polynomials of compact simple Lie groups of four classe were first determined by R. Brauer. Pontrjagin investigated the homological properties of these manifolds and determined their homology basis. *The second class* of symmetric Riemannian manifolds are those whose fundamental groups are simple. Among the manifolds of these kinds there exist those which are at the same time *algebraic varieties.* The Betti numbers and the correspondingly homology basic of these manifolds were determined by C. Ehresmann. (The manifolds $S(n)$, $C(n)$, $A(n, k)$)[1]. In this paper I will give the complete table of Betti numbers of compact symmetric Riemannian manifolds by the methode E. Cartan.

Notations.

R_n: n-dimensional real vector space.

P_n: n-dimensional complex vector space.

k: The field of all real numbers, K: That of all complex numbers.

E or E_n: n-dimensional identity matrix, $\delta(ij)$ or δ_{ij}: $E=\delta(ij)$.

I or I_n: The skew matrix $\begin{Vmatrix} & E_{n/2} \\ -E_{n/2} & \end{Vmatrix}$, $\varepsilon(ij)$ or ε_{ij}: $I=\varepsilon(ij)$.

$US(n)$, $U(n)$, $USp(n)$, $O(n)$: unitary unimodular group, unitary group, unitary-symplectic group and real proper orthogonal group respectively.

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(1) These are manifolds in which the Betti numbers of odd dimensions are all zero, see E. Cartan, Selecta, p. 103; Ehresmann, [3].