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A boundary value problem of some special ordinary differential equations of the second order.

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§ 1. Statement of the problem.

1. Among various results concerning the behaviour of geodesics in the large, there are many theorems which can be stated without the concept of length. They seem to be only properties in the large of integral curves of a system of differential equations of the second order. Hence, some of them may be generalized to the geometry of paths in the large.

Now, the first problem in the theory of geodesics in the large is that "Given any two points on a surface, can they always be bound by a geodesic?" From the point of view stated above, there then arises the following problem: Given any two points in a plane, can they always be bound by a path? A path is, by definition, an integral curve of a system of ordinary differential equations of the second order of the following type

$$\begin{aligned} \ddot{x} &= A_1\dot{x}^2 + 2B_1\dot{x}\dot{y} + C_1\dot{y}^2 , \\ \ddot{y} &= A_2\dot{x}^2 + 2B_2\dot{x}\dot{y} + C_2\dot{y}^2 , \end{aligned} \tag{1}$$

where dots denote derivatives with respect to a parameter t, and A_1, B_1,... ..., C_2 denote continuous functions of x and y. Putting $x=x^1$, $y=x^2$, the set of equations (1) are usually written as

$$\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k = 0 , \quad (i, j, k=1, 2) \tag{2}$$

Γ^i_{jk} being called parameters of an affine connexion.

The answer of the problem is in general negative. But, it is desirable to know in what manner it becomes impossible, in other words, the behaviour of integral curves.

In this paper we shall confine ourselves to the simplest case where A_1, B_1, ..., C_2 are all real constants. Our result may be stated as follows:

Theorem. *Let there be given a system of differential equations of the form* (1) *with real constant coefficients A_1, B_1, ..., C_2. Then they can be classified into two types. For one of them any two points in plane can be*