J. Math. Soc. Japan
Vol. 45, No. 3, 1993

On a class of hypoelliptic differential operators with double characteristics

Dedicated to Professor Mutsuhide Matsumura on his 60th birthday in 1991

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(Received Nov. 18, 1991)
(Revised May 21, 1992)

Introduction and results.

This paper is devoted to the study of *hypoellipticity* for second-order degenerate *elliptic* differential operators $P(x, D)$ with real coefficients on $\boldsymbol{R}^n$ of the form:

$$P(x, D)=\frac{\partial^2}{\partial x_1^2}+\sum_{i,j=2}^{n}\frac{\partial}{\partial x_i}\left(a^{ij}(x)\frac{\partial}{\partial x_j}\right)+\sum_{i=1}^{n}b^i(x)\frac{\partial}{\partial x_i}+c(x),$$

where:

1) The a^{ij} are the components of a C^∞ symmetric contravariant tensor of type $\binom{2}{0}$ on $\boldsymbol{R}^n$, and

$$\sum_{i,j=2}^{n}a^{ij}(x)\xi_i\xi_j \geqq 0 \qquad \text{on } T^*(\boldsymbol{R}^n).$$

Here $T^*(\boldsymbol{R}^n)$ is the cotangent bundle of $\boldsymbol{R}^n$.

2) $b^i \in C^\infty(\boldsymbol{R}^n)$.

3) $c \in C^\infty(\boldsymbol{R}^n)$.

Let u be a distribution on an open subset Ω of $\boldsymbol{R}^n$. The singular support of u, denoted by sing supp u, is the complement of the largest open subset of Ω on which u is of class C^∞. A differential operator $P(x, D)$ is said to be *hypoelliptic* in Ω if it satisfies the condition:

$$\operatorname{sing\,supp} u=\operatorname{sing\,supp} Pu \qquad \text{for all} \quad u \in \mathcal{D}'(\Omega).$$

This condition is equivalent to the following:

This research was partially supported by Grant-in-Aid for General Scientific Research (No. 03640122), Ministry of Education, Science and Culture.