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The exterior non-stationary problem for the Navier-Stokes equations in regions with moving boundaries

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1. Introduction.

We consider the motion of a viscous incompressible fluid in an exterior domain with moving boundaries, in other words we have to deal not with a space-time cylinder but with a noncylindrical domain in $R^3\times[0, T]$. To be more precise, we consider a domain

$$\Omega_T=\bigcup_{0\leq t\leq T}\Omega(t)\times\{t\}$$

where each $\Omega(t)$ is the exterior of a bounded connected domain $\Omega^c(t)$ in R^3, and $T>0$ is a finite number.

The exterior problem for the Navier-Stokes equations consists of finding in the region Ω_T exterior to a closed bounded surface, the velocity u and the pressure p which together solve the system (1.1) given below, and are such that the velocity assumes a given value on the surface, for $|x|\rightarrow\infty$, and in $t=0$.

The motion of the fluid in Ω_T is governed by the following equations

$$\partial_t u-\mu\Delta u+u\cdot\nabla u=f-\nabla p\,,\qquad \nabla\cdot u=0\qquad \text{in } \Omega_T \tag{1.1}$$

where $\partial_t=\partial/\partial t$, $u=u(t)=(u_1(x, t), u_2(x, t), u_3(x, t))$ is the velocity, $p=p(t)=p(x, t)$ the pressure, $f=f(t)=(f_1(x, t), f_2(x, t), f_3(x, t))$ the external force, and μ the viscosity. We take the motion of the fluid at $t=0$ to be known, hence $u(x, 0)=u_0$ is a prescribed vector field in $\Omega(0)$. Let

$$\Gamma_T=\bigcup_{0\leq t\leq T}\Gamma(t)\times\{t\}$$

where $\Gamma(t)$ is the boundary of $\Omega^c(t)$. Throughout the paper we suppose that Γ_T is smooth enough and $\Omega(t)$ does not change its topological type as t increases over $[0, T]$. The classical formulation of the problem is the velocity u and the pressure p to satisfy (1.1) and the initial boundary conditions

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