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Scattering theory by Enss' method for operator valued matrices: Dirac operator in an electric field

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§1. Introduction.

The geometric method of Enss [1] for differential operators in $L^2(\boldsymbol{R}^n)$ is now well established. In this article we extend it to a class of differential operators in $[L^2(\boldsymbol{R}^n)]^m$, $m\geqq 2$. Our class includes the Dirac operator with an electric field in $[L^2(\boldsymbol{R}^3)]^4$; for details refer to example 2.2.

Spectral theory and scattering theory were considered for the operator $P^2/2+W_s$ on $L^2(\boldsymbol{R}^n)$ where W_s is a short range potential in [**1, 2, 3, 4, 5**]. For general operators of the form $h_0(P)+W_s$ on $L^2(\boldsymbol{R}^n)$ with $h_0(\infty)=\infty$ refer to [**6, 7**]. For a hint of developing the geometric method for opepators in $[L^2(\boldsymbol{R}^n)]^m$ refer to [**6**].

For the operator $P^2/2+W_s$ the boundedness of the eigenvalues is proved in [**8**].

For the operator $P^2/2+W_s+W_L(Q)$ where, now, W_L is a smooth long range local potential the theory is developed in [**9, 10, 11, 12, 13, 14, 15, 16, 17, 18**]. General operator of the form $h_0(P)+W_s(Q, P)+W_L(Q, P)$ with $h_0(\infty)=\infty$ is considered in [**19**]. For an account of all these results see [**20, 21**].

For a class of operators of the form $h_0(P)+W_s$ where h_0 need not have any limit at ∞ the geometric theory is developed in [**22, 23**].

Finally we sketch the contents of the article. In §2 we state the assumptions on the operator H and state the main theorem we intend to prove. In §3 we reproduce some technical theorems from [**19**]. These will be repeatedly used in §4 and §5. Existence of the wave operator is proved in §4 where as in §5 we prove asymptotic completeness.

§2. Statement of the result.

On the free and perturbed operators H_0 and H on the Hilbert space $[L^2(\boldsymbol{R}^n)]^m$, $n, m\geqq 1$ we make the following set of assumptions A1, A2, $\cdots$, A9.

A1. H_0: $\boldsymbol{R}^n\rightarrow\mathcal{M}_m(\boldsymbol{C})$, where $\mathcal{M}_m(\boldsymbol{C})$ is the space of all $m\times m$ matrices with entries from the complex numbers $\boldsymbol{C}$, is a C^∞ function and for each ξ in $\boldsymbol{R}^n$ the