Tôhoku Math. Journ.
27(1975), 445-451.

CONFORMALLY FLAT RIEMANNIAN MANIFOLDS ADMITTING A TRANSITIVE GROUP OF ISOMETRIES II

HITOSHI TAKAGI

(Received July 31, 1974)

1. Introduction. Let M be an n-dimensional conformally flat Riemannian manifold which admits a transitive group of isometries. In [1], the author proved that M is isometric to one of the homogeneous Riemannian manifolds of following types:

(I) $$S^n(K)/\Gamma\ ,\quad E^n/\Gamma\ ,\quad H^n(-K)\ ,$$

(II) $$(S^r(K)/\Gamma)\times H^{n-r}(-K)\ ,\quad 2\leqq r\leqq n-2\ ,$$

(III) $$(E^1/\Gamma)\times H^{n-1}(-K)\ ,$$

(IV) $$(S^{n-1}(K)\times E^1)/\Gamma\ ,$$

where $S^m(K)$, E^m and $H^m(-K)$ denote a Euclidean m-sphere of radius $K^{-1/2}$, a Euclidean m-space and a hyperbolic m-space of curvature $-K$ respectively. N/Γ denotes a quotient space, where Γ is a group of isometries of N acting freely and properly discontinuously. And $\times$ denotes a Riemannian product.

J. A. Wolf (see [2]) classified the homogeneous Riemannian manifolds of the forms $S^n(K)/\Gamma$ and E^n/Γ. Thus, the problem left to us is to determine the groups Γ appearing in (IV). In [1], the author proved a few theorems about the structure of Γ. Making use of the theorem, we shall classify the manifolds of type IV completely.

2. Structure of Γ. We consider $S^{n-1}(K)$ as the set of vectors of norm $K^{-1/2}$ in a Euclidean vector space $\boldsymbol{R}^n$. Then, the group of all isometries of $S^{n-1}(K)$ is the orthogonal group $O(n)$. Let $\boldsymbol{Q}$ and $\boldsymbol{Q}'$ denote the algebra of real quaternions and the multicative group of unit quaternions respectively. $\boldsymbol{Q}'=\{a=a_1+a_2i+a_3j+a_4k;\ \sum_{s=1}^4 a_s^2=1,\ a_s\in\boldsymbol{R}\}$ has an $SO(4l)$-representation ρ defined by

$$\rho\colon a\longrightarrow\begin{pmatrix}\overbrace{A\quad\quad}^{l}&&\\&\ddots&\\&&A\end{pmatrix},\quad\text{where}\quad A=\begin{pmatrix}a_1&-a_2&-a_3&-a_4\\a_2&a_1&-a_4&a_3\\a_3&a_4&a_1&-a_2\\a_4&-a_3&a_2&a_1\end{pmatrix}.$$