ON THE STRUCTURE OF RIEMANNIAN SPACES WHOSE HOLONOMY GROUPS FIX A NULL-SYSTEM.*)

By

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§ 1. Introduction.

Let V_n be an n-dimensional Riemannian space with positive definite line element

(1.1) $$ds^2 = g_{jk}(x)dx^j dx^k$$

in each of its coordinate neighbourhoods. At each point P of V_n, vectors in the tangent Euclidean space $E_n(P)$ constitute an n-dimensional vector space $R_n(P)$. If we understand contravariant vectors X^i and covariant vectors U_i in $R_n(P)$ as if they were homogeneous point and hyperplane-coordinates, the $R_n(P)$ may be regarded as an $(n-1)$-dimensional projective space P_{n-1}.

Now let S_{ij} be a skew symmetric covariant tensor at P with non-vanishing determinant. (Accordingly we suppose hereafter that n is an even integer). The correspondence

(1.2) $$X^i \to S_{ij}X^j$$

may be regarded as a null-system in P_{n-1}. On account of this fact we shall call hereafter such $S = (S_{ij})$ as a *null-system* at P in V_n.

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**) On the 27 th of June 1948, Mr. Iwamoto died of consumption in the city of Okayama. He was born on the 10th, February 1923. In March 1942, he graduated from the mathematical Institute, Tôkyô University. Since 1944, he worked at the mathematical Institute of Nagoya University. He studied on differential geometries especially higher order geometry, symmetric spaces and theory of holonomy groups etc.. Some of his researches were published in Japanese Journal of Mathematics and Mathematica Japonicae.

His death was indeed a great loss in our mathematical circle. About half year before his death, he showed us some of his manuscripts on the theory of holonomy groups. Putting the contents of one of his manuscripts in order, adding proofs and translating in English, we publish it here. Of all contents of this paper the translator takes the responsibility.

(Translator: S. Sasaki).