NOTES ON FOURIER ANALYSIS (XXVI): SOME NEGATIVE EXAMPLES IN THE THEOREY OF FOURIER SERIES*)

By

SHIN-ICHI IZUMI

Introduction. W. Randels[1] has proved the following Theorem.

THEOREM A. *There is a function $f(t) \in L^2$ such that*

1^0. $\int_0^t |\varphi_x(u)| du \neq o(t)$, $\varphi_x(t) = f(x+t) + f(x-t) - 2f(x)$,

2^0. *the series*

$$\sum_{n=2}^{\infty} (a_n \cos nx + b_n \sin nx)/\sqrt{\log n} \tag{0.1}$$

converges, where

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt). \tag{0.2}$$

R. E. A. C. Paley[2] has proved

THEOREM B. *There is an integrable function $f(t)$ such that*

1^0. $\int_0^t \varphi_x(u) du \neq o(t)$,

2^0. *the Fourier series* (0.2) *of $f(t)$ converges at $t = x$.*

As a generalization of Theorem A we prove that

THEOREM 1. *There is a bounded function $f(t)$ such that*

1^0. $\int_0^t |\varphi_x(u)| du \neq 0$,

2^0. *the Fourier series* (0.2) *of $f(t)$ converges at $t = x$.*

We prove also the following theorem containing Theorem A and B. That is,

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*) Received Oct. 1, 1949.

1) W. Randels, Bull. Am. Math. Soc., 46 (1940).

2) R. E. A. C. Paley, Proc. Cambridge Phil. Soc., 26 (1930).