ON NON-CONNECTED MAXIMALLY ALMOST PERIODIC GROUPS*)

By

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All groups which we shall consider in this note are locally compact. If G is a group, let G^0 be the connected component of the identity in G. Under the restriction of connectedness a necessary and sufficient condition for G to be maximaly almost periodic (m.a.p. for simplicity) is, as well known, that G is decomposed into a direct product of a compact group and a vector group. In the present note, we shall consider the non-connected case where however G/G^0 is compact. Firstly, an analogous proposition as above is not valid in this case (cf. Remark 1), and only the space of G is decomposed into the topological direct product of the space of a compact subgroup and that of a closed subgroup which is isomorphic with a vector group (cf. Remark to Theorem A). Moreover, the criterion for the maximally almost periodicity of G is given by that of G^0. In fact, if G^0 is m.a.p., G is also m.a.p. (Theorem B).

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LEMMA 1[1]. *Let N be a closed normal subgroup of G. If N is a vector group and G/N is a compact group, G contains a compact subgroup K such as $K\cdot N = G$, $K \cap N = \{ e \}$*[2].

THEOREM A. *Let G/G^0 be compact and G^0 be m.a.p. Then G contains a compact group K and a vector group N such as*

$$G = K\cdot N, \qquad K \cap N = \{ e \}.$$

PROOF. As G^0 is connected and m.a.p., $G^0 = K_1 \times N$, where K_1 is a compact normal subgroup of G and N is a vector group. As K_1 is compact, $G^0/K_1\,(\simeq N)$ is a closed normal subgroup of G/K_1 and $(G/K_1)/(G^0/K_1)$ is ccmpact. By Lemma 1, there exists a compact subgroup K of G containing K_1 such as

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1) This lemma is obtained by K. Iwasawa. For the proof see Iwasawa [2], esp. Lem. 3.8.

2) In this note e denotes the identity of groups.