BEURLING'S THEOREM ON EXCEPTIONAL SETS*)

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1.

1. Let E be a bounded Borel set of points on z-plane. We distribute a positive mass $d\mu(a)$ of total mass 1 on E and let

$$u(z) = \int_E \log \frac{1}{|z-a|} \, d\mu(a), \qquad (\mu(E) = 1),$$

then $u(z)$ is harmonic outside of E. Let V_μ be the upper limit of $u(z)$ for $|z| < \infty$ and $V = \inf_\mu V_\mu$, then $C(E) = e^{-V}$ is called the logarithmic capacity of E. Hence if $C(E) > 0$, i.e. $V < \infty$, then we can distribute a positive mass $d\mu$ on E, such that $V_\mu < \infty$.

Evans[1] proved the following theorem, which we use in the proof of Theorem 5.

LEMMA 1. (EVANS.) *Let E be a bounded closed set of logarithmic capacity zero on z-plane, then we can distribute a positive mass of total mass 1 on E, such that $u(z)$ tends to $+\infty$, when z tends to any point of E.*

Beurling[2] proved the following important theorems:

THEOREM 1. (BEURLING.) *Let $w = f(z)$ be regular in $|z| < 1$ and the area A on w-plane, which is described by $w = f(z)$ $(|z| < 1)$ be finite, i.e.*

$$A = \iint_{|z|<1} |f'(re^{i\theta})|^2 \, r dr d\theta < \infty,$$

then the set E of points $e^{i\theta}$ on $|z| = 1$, such that

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1) G. C. Evans: Potentials and positively infinite singularities of harmonic functions. Monatshefte für Math. u. Phys. **43** (1936). Evans proved for Newtonian potentials and the proof can be easily modified in the case of logarithmic capacity. This is done by K.Noshiro in his paper: Contributions to the theory of the singularities of analytic functions. Jap. Jour. Math. 19 No.4 (1948).

2) Beurling: Ensembles exceptionelles. Acta Math. **72** (1940).