ON THE KLEIN'S GEOMETRY*)

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Introduction

Klein's definition of geometry can be stated as follows. Suppose that a point-set M and a group G of 1-1 transformations, each of which maps M onto itself, are given. An arbitrary subset A of M is said to be a figure and a figure A is said to be congruent to a figure B when B is a image of A under a transformation of G. We denote this relation by $A \equiv B$. Then, as G is a group, the concept of congruence satisfies the following three conditions (equivalence relations):

1) Every figure is congruent to itself.

2) If a figure A is congruent to a figure B, then B is congruent to A.

3) If a figure A is congruent to a figure B and B to C, then A is congruent to C.

It therefore enables us to divide the figures in M into mutually disjoint congruence classes. It is the Klein's geometry to study the common properties of all figures of the same classes, namely the properties which are invariant under all transformations of G.

But a kind of geometry will be constructed, if the set of transformations Λ satisfies the above conditions, even if it does not form a group.

Thus there arises a question that whether Λ forms a group or not as an implication of the fact that the concept of congruence satisfies the above three conditions.

This question was pointed out by Professor S. Sasaki, but there exists a very simple example such that Λ is not a group.

On the other hand, the developments of the theories of projective, affine, Euclidean and Möbius' geometries are based on the fact that G's are groups of projective, affine, Euclidean, and Möbius' transformations respectively. And it will be hard to expect fine theories, if the concept of congruence satisfies only the above three conditions.

Therefore it will be necessary for the foundation of Klein's geometry to find conditions other than the above three which express the conditions of congruence, in order that Λ forms a group.

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