ON SUBPROJECTIVE SPACES, I

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0. Introduction. In an n-dimensional affine space A_n, when geodesics are represented by $n-2$ homogeneous linear equations with respect to coordinates and another equation for a suitable coordinate system, B. Kagan[2] called A_n a *subprojective space.*

The $n-2$ homogeneous linear equations represent a two-dimensional surface and may be written in the form

$$\gamma^h x^h = \alpha^h x^{n-1} + \beta^h x^n \qquad (h = 1, 2, \cdots n-2), \tag{0.1}$$

where α^h and β^h are constants. Consequently, in the subprojective space, geodesics lie on two-dimensional surfaces whose equations are given by the form (0.1) for a suitable coordinate system. From (0.1) we find that the affine connection $\Gamma^\lambda_{\mu\nu}$ takes the form

$$\Gamma^\lambda_{\mu\nu} = \varphi_\mu \delta^\lambda_\nu + \varphi_\nu \delta^\lambda_\mu + \varphi_{\mu\nu} x^\lambda, \tag{0.2}$$

where φ_μ and $\varphi_{\mu\nu}$ are any covariant vector and symmetric tensor respectively.

Conversely, if the affine connection is given by (0.2) for a suitable coordinate system, we can conclude that A_n is a subprojective space.

P. Rachevsky[5] proved that, when φ_μ is a gradient vector, affine connection (0.2) is reducible to the form

$$\bar{\Gamma}^\lambda_{\mu\nu} = \bar{u}_{\mu\nu} \bar{x}^\lambda$$

by a transformation of coordinates

$$\bar{x}^\lambda = e^\varphi x^\lambda,$$

where $\varphi_\mu = \dfrac{\partial\varphi}{\partial x^\mu}$. The coordinate system $\bar{x}^\lambda$ is called the *canonical* system.

However, in a subprojective Riemannian space, since it is concluded from (0.2) that φ_μ is a gradient vector, Christoffel symbols of the second kind may be written as

$$\left\{ {\lambda \atop \mu\nu} \right\} = u_{\mu\nu} x^\lambda \tag{0.3}$$

with respect to the canonical coordinate system.

From (0.3) P. Rachevsky introduced relations, as a necessary and sufficient condition that a Riemannian space be subprojective,

$$\begin{aligned} &(\mathrm{A}) \quad R_{\lambda\mu\nu\omega} = T_{\lambda\omega} g_{\mu\nu} + T_{\mu\nu} g_{\nu\omega} - T_{\lambda\nu} g_{\lambda\omega} - T_{\mu\omega} g_{\lambda\nu}, \\ &(\mathrm{A}') \quad T_{\mu\nu;\omega} - T_{\mu\omega;\nu} = 0, \\ &(\mathrm{B}) \quad T_{\mu\nu} = \rho g_{\mu\nu} + \rho_\mu \sigma_\nu, \end{aligned} \tag{0.4}$$

where

$$T_{\mu\nu} = \frac{1}{n-2}\left(R_{\mu\nu} - \frac{R}{2(n-1)} g_{\mu\nu}\right), \tag{0.5}$$