ON MAPPINGS FROM COMPLEXES INTO THE COMPLEX PROJECTIVE SPACE

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1. Introduction. L. Pontrjagin [2] classified the mappings from any three-dimensional complex into the 2-sphere, and also classified the algebraically inessential mappings from any arbitrary dimensional complex into the 2-sphere. In this paper, we shall treat a generalization of his results. Namely, for two mappings f, g from the $(2n+1)$-dimensional complex in the complex n-dimensional complex projective space, whose partial mappings are the same in the 2-skeleton, an integral $(2n+1)$-cocycle, which we shall call the *separation obstruction* $d^{2n+1}(f, g)$, can be defined; and the necessary and sufficient condition that they are homotopic is that there exists an integral 1-cocycle w^1 such that $d^{2n+1}(f, g) \sim 2^n w^1 \cdot (f^* s^2)^n$, where the product of the right-hand side of this cohomology means the cup product of cocycles based on the usual products of integers of coefficients. When $n = 1$, this result is nothing but the theorem of L. Pontrjagin [2] and proved by N. E. Steenrod [3] by another method. Here we shall prove the result by the method of N. E. Steenrod.

Secondly, we shall show, that the classes of algebraically inessential mappings from the arbitrary dimensional complex. whose 1-cohomology group with the integral coefficient vanishes, into the complex projective space, are in 1-1 correspondence to the classes of mappings from that complex into the $(2n+1)$-dimensional sphere. But the author can not avoid here the cohomological assumption in the case $n > 1$. We shall prove this last half by the theory of fibre bundles.

2. Complex projective space. Let an oriented $(2n+1)$-sphere S^{2n+1} be represented by the coordinates of the complex numbers $(z_0, z_1, \dots, z_n)$ as follows:

$$\sum_{k=0}^{n} z_k \bar{z}_k = 1.$$

Now, let us call two points $(z_0, \dots, z_n)$ and $(z'_0, \dots, z'_n)$ on S^{2n+1} to be *equivalent* provided that there exists a real number θ such that:

$$z_k = \exp(\sqrt{-1}\,\theta) z'_k \quad (k = 0, 1, \dots, n).$$

Then the factor space defined by this equivalence relation, is the *complex projective space* $P(n)$; and it is well known [4; p. 108] that S^{2n+1} is an orientable circle bundle[1] over $P(n)$, and that the canonical mapping $p_n : S^{2n+1} \to P(n)$ is the

1) As for fiber bundles, we shall mainely use the definitions and the notations of [4].