THE IRREDUCIBLE DECOMPOSITIONS OF THE MAXIMAL HILBERT ALGERAS OF THE FINITE CLASS

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The notion of the Hilbert algebras was introduced by H. Nakano [8] and W. Ambrose [1], independently, and their relations were studied by O. Takenouchi [13]. Recently R. Godement [4] has introduced the double unitary representations of a unimodular locally compact group by the central Radon measure of the positive type, but their research is clearly reduced to the one of the maximal Hilbert algebras.

The object of the present paper is to obtain the decompositions of the maximal Hilbert algebras of the finite class by the method due to Godement in the theory of the double unitary representations of a group [4]. But it is remarkable to obtain the irreducibility of the decompositions without any separability condition, and these results may be suggestive to the central decompositions of the arbitrary W^*-algebras.

As we have extended the notion of the $\natural$-oparation, introduced by J. Dixmier [2], to the arbitrary W^*-algebras in the previous papers [10, 11], we can decompose the arbitrary maximal Hilbert algebra in this way, but we cannot yet prove the irreducibility in this case. Therefore, we shall discuss these problems in the form of the extension of the Plancherel formula to a unimodular group in the next paper [12].

Finally, some related problems are announced by Takenouchi [13], but it seems to be necessary to introduce the separability condition there.

1. Fundamental properties of Hilbert algebras.

Following Nakano [8],

DEFINITION 1.1. A linear manifold $\mathfrak{A}$ of a Hilbert space $\mathfrak{H}$ is called a *Hilbert algebra*, if (1) $\mathfrak{A}$ is dense in $\mathfrak{H}$; (2) $\mathfrak{A}$ is an algebra with complex coefficients; (3) for any $a \in \mathfrak{A}$ there is an adjoint element $a^* \in \mathfrak{A}$ such that

(1.1) $$< ab, c > = < b, a^*c >, \quad < ba, c > = < b, ca^* >;$$

(4) for any $a \in \mathfrak{A}$ there exists a positive number α_a such that $\| ax \| \leqq \alpha_a \| x \|$ for all $x \in \mathfrak{A}$; (5) by (1) and (4), for every $a \in \mathfrak{A}$ we obtain uniquely a bounded linear operator L_a on $\mathfrak{H}$ such that $L_a x = ax$ for all $x \in \mathfrak{A}$. (6) For an element $x \in \mathfrak{H}$, if $L_a x = 0$ for all $a \in \mathfrak{A}$, then we have $x = 0$.

DEFINITION 1.2. A Hilbert algebra $\mathfrak{A}$ is said to be *maximal*, if there is no *extension* except itself, that is, a Hilbert algebra containing $\mathfrak{A}$ as a subalgebra.

As can be easily verified, we have ([8; § 1])