ABSOLUTE CESÀRO SUMMABILITY OF ORTHOGONAL SERIES

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Introduction

For a series $\sum_{n=0}^{\infty} a_n$ denote by σ_n^α the n-th Cesàro mean of order α $(\alpha > -1)$, i.e.,

$$\sigma_n^\alpha = \frac{1}{A_n^\alpha} \sum_{k=0}^{n} A_{n-k}^\alpha a_k$$

where

$$A_n^\alpha = \binom{n+\alpha}{n} = \frac{(\alpha+1)(\alpha+2)\ldots(\alpha+n)}{n!} \simeq \frac{n^\alpha}{\Gamma(\alpha+1)} .$$

If the series

$$\sum_{n=0}^{\infty} |\sigma_{n+1}^\alpha - \sigma_n^\alpha|$$

converges, we say that the series $\sum a_n$ is absolutely Cesàro summable with order α or briefly summable $|C, \alpha|$.

Various theorems concerning this summability of orthogonal series and of Fourier series were obtained by many authors. One of them is the following F. T. Wang's theorem [5]

THEOREM A. (i) *If the series*

$$\sum_{n=1}^{\infty} (a_n^2 + b_n^2) (\log n)^{1+\epsilon} \tag{1}$$

is convergent for some $\varepsilon > 0$, *then the trigonometric series*

$$\sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{2}$$

is summable $|C, \alpha|$ *for almost every* x, *where* $\alpha > 1/2$;

(ii) *If the series*

$$\sum_{n=1}^{\infty} (a_n^2 + b_n) (\log n)^{2+\epsilon} \tag{3}$$

is convergent for some $\varepsilon > 0$, *then the series* (2) *is summable* $|C, 1/2|$ *for almost every* x.

(iii) *If* $0 < \alpha < 1/2$, *and if the series*