NOTE ON CONSERVATIVE ALGEBRAIC FUNCTION FIELDS

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J. Tate's formula of genus reduction in his article "Genus reduction in purely inseparable extension of algebraic function fields", Proc. Amer. Math. Soc. (1952), gives a solution to the problem to characterize conservative algebraic function fields, stated in E. Artin's "Algebraic numbers and algebraic functions I", New York (1951), which we quote as A. N. F. in the following, but we discuss in the present article the problem directly on the base of the Chapter XV of A. N. F., especially on the Theorem 20 there.

Though our results follows also from above Tate's formula without any difficulties, it seems to the writer that our treatment based on a $\mathfrak{p}$-adic number theoretical lemma (Lemma 4 in the following) has some interest.

THEOREM 1. *Let k be an algebraic function field of transcendental degree 1 with coefficient field k_0 of characteristic p $(\neq 0)$. Suppose that there exists an element x of k, not belonging to k_0, such that the rank n of k over $k_0(x)$ is not divided by p. Then k is conservative, if and only if every prime ideals of $k_0[x]$ generated by polynomials of x^p with coefficients in k_0 does not ramify, that is, any irreducible polynomials of x with coefficients in k_0 dividing the discriminant of the principal order of k over $k_0[x]$ are not polynomials of x^p with coefficients in k_0.*

PROOF. To prove the Theorem 1, it is sufficient to show that the Theorem holds when k_0 is separably algebraically closed (i. e. when every separably algebraic elements over k_0 are involved in k_0 itself). Because, when k_0 is not so, we take the separably algebraic closure $\bar{k}_0^s$ of k_0, the field consisting of every separably algebraic elements over k_0, and we extend the coefficient field k_0 of k to $\bar{k}_0^s$, denote k_0k by k'; then clearly the genus of k' is equal to that of k, and k' is conservative, if and only if k is conservative; on the other hand, as it holds clearly that

$$[K: \bar{k}_0^s(x)] = [k: k_0(x)],$$

the same x in k satisfies the assumption of the Theorem for k' with coefficient field $\bar{k}_0^s$; and as irreducible polynomials of x in $k_0[x]$ which are polynomials of x^p with coefficients in k_0 resolve into products of different linear polynomials of x^p with coefficients in $\bar{k}_0^s$, there exists a prime ideal in $\bar{k}_0^s[x]$ satisfying the conditions of the Theorem for k', if and only if there exists a prime ideal of $k_0[x]$ satisfying that for k.