HOMOLOGY GROUPS IN CLASS FIELD THEORY

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Recently, J. Tate[1] has given an interesting theorem that the higher dimensional cohomology groups $H^r(G, A)$ occurring in class field theory, i.e. A: the multiplicative group of nonzero elements in a $\mathfrak{p}$-adic field or the idèle class group in an algebraic number field, G: the galois group, are canonically isomorphic to the integral cohomology groups $H^{r-2}(G, Z)$ for every $r > 2$. It was stated, without details, that one can introduce negative dimensional cohomology groups and the isomorphisms:

$$H^{r-2}(G, Z) \cong H^r(G, A)$$

are valied for every dimensions. Moreover, the isomorphism

$$H^{-2}(G, Z) \cong H^0(G, A)$$

from $H^{-2}(G, Z) =$ commutator factor group of G, to $H^0(G, A) =$ idèle norm residue class group, is the reciprocity law mapping.

I shall show in this note that if we put

$$H^{-r}(G, A) = H_{r-1}(G, A) \qquad (r = 1, 2, \ldots)$$

where $H_{r-1}(G, A)$ is the $(r-1)$-dimensional homology group, then all statements of Tate hold. Moreover, the isomorphism

$$H^{-3}(G, Z) \cong H^{-1}(G, A)$$

is the isomorphism theorem of H. Kuniyoshi[2] in the theory of T. Tannaka[3] concerning the "*Hauptgeschlechtssatz im Minimalen*".

1. Let G be a finite group, A a G-module, we now define boundary and coboundary operators ∂, δ for the module of q-chains[4] $C_q(G, A)$ of G with value in A:

$$\partial f(x_1, \ldots, x_{q-1}) = \sum_{x \in G} x^{-1} f(x, x_1, \ldots x_{q-1})$$

1) J. TATE, Higher dimensional cohomology groups of class field theory. Ann. of Math., 56(1952), 294-297.

2) H. KUNIYOSHI, On a certain group concerning the p-adic number field, Tohoku Math. Journ., 1(1950), 186-193, Theorem 2.

3) T. TANNAKA, Some remarks concerning p-adic number field, Journ. of Math. Soc. of Japan, 3(1951), 252-257, Theorem 2.

4) q-chain is a function of q-variable in G to A; therefore identical with q-cochain. For infinite group G, one must restrict the function to the class that are $\neq 0$ only for some finite systems $(x_1, \cdots, x_q)$ of elements in G.