TWO THEOREMS ON THE RIEMANN SUMMABILITY

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(Received November 25, 1953)

1. The series $\sum_{\nu=1}^{\infty} a_\nu$ is said (R_1)-summable to zero if the series

$$F(t) = \sum_{\nu=1}^{\infty} \frac{s_\nu}{\nu} \sin \nu t, \tag{1}$$

where $s_n = \sum_{\nu=1}^{n} a_\nu$, converges in some interval $0 < t < t_0$, and if $F(t)$ tends to zero as t tends to zero.

The series $\sum_{\nu=1}^{\infty} a_\nu$ is said $(R, 1)$-summable to zero if the series

$$G(t) = \sum_{\nu=1}^{\infty} a_\nu \frac{\sin \nu t}{\nu t} \tag{2}$$

converges in some interval $0 < t < t_0$, and if $G(t)$ tends to zero as t tends to zero.

Recently, one of the present authors [2] proves the following theorem;

THEOREM A. *Suppose that*

$$\sum_{\nu=1}^{n} s_\nu = o(n^\alpha),$$

$$\sum_{\nu=n}^{\infty} \frac{|a_\nu|}{\nu} = O(n^{-\alpha}),$$

where $0 < \alpha < 1$. *Then the series* $\sum_{\nu=1}^{\infty} a_\nu$ *is* (R_1)*-summable to zero.*

THEOREM B. *Under the assumptions of Theorem A, the series* $\sum_{\nu=1}^{\infty} a_\nu$ *is* $(R, 1)$*-summable to zero.*

The object of this paper is to generalize the above theorems.

THEOREM 1. *Let* s_n^β *be the* (C, β)*-sum of* $\sum_{n=1}^{\infty} a_n$. *Then, if*

$$s_n^\beta = o(n^{\beta\alpha}), \tag{3}$$

and

$$\sum_{\nu=n}^{\infty} \frac{|a_\nu|}{\nu} = O(n^{-\alpha}), \tag{4}$$