ON THE GENERATION OF STRONGLY ERGODIC SEMI-GROUPS OF OPERATORS II*)

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1. Introduction. This paper is concerned with the problem of determining necessary and sufficient conditions that a linear operator is the infinitesimal generator of a semi-group of bounded linear operators. The first results in this direction were published independently by E. Hille [2][1] and K. Yosida [10] for semi-groups of operators satisfying the following conditions :

(c_1) *$T(\xi)$ is strongly continuous at $\xi = 0$,*

(c_2) *$\|T(\xi)\| \leqq 1 + \beta\,\xi$ for sufficiently small ξ, where β is a constant.*

Their results were later generalized to semi-groups of operators satisfying only the condition (c_1) by R. S. Phillips [8] and the present author [4], independently. Further this result has been generalized to strongly measurable semi-groups of operators by W. Feller [1]. R. S. Phillips [9] and the present author [5] have recently given necessary and sufficient conditionsthat a given operator generates a semi-group of class $(1, A)$ or of class $(1, C_1)$. In the present paper we give a necessary and sufficient condition that a given operator generates a semi-group of class $(0, A)$ or of class $(0, C_\alpha)$.

2. Definitions and preliminary theorems. Let $\{T(\xi);\ 0 \leqq \xi < \infty\}$ be a semi-group of operators satisfying the following conditions:

(a) For each ξ, $0 \leqq \xi < \infty$, $T(\xi)$ is a bounded linear operator from a complex Banach space X into itself and

$$T(\xi + \eta) = T(\xi)T(\eta) \qquad \text{for } \xi, \eta \geqq 0, \tag{2.1}$$
$$T(0) = I\,(= \text{the identity}).$$

(b) $T(\xi)$ is strongly measurable on $(0, \infty)$ (see [2, Definition 3.3.2]).

(c)
$$\int_0^1 \|T(\xi)x\|\, d\xi < \infty \qquad \text{for each } x \in X.$$

A consequence of (a) and (b) is that $T(\xi)$ is strongly continuous for $\xi > 0$ (see [3] and [7]). If $T(\xi)$ satisfies the condition

(d)
$$\lim_{\lambda\to\infty} \lambda \int_0^\infty e^{-\lambda\xi}\, T(\xi)x d(\xi) = x \qquad \text{for each } x \in X,$$

then $T(\xi)$ is said to be of *class* $(0, A)$. If, instead of (d), $T(\xi)$ satisfies the

*) This paper is the continuation of the paper with the same title in this Journal, vol. 6 (1954).

1) Numbers in brackets refer to the references at the end of this paper.