ON THE CESÀRO SUMMABILITY OF FOURIER SERIES

MASAKITI KINUKAWA

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1. Introduction. Let $f(t)$ be an integrable function periodic with period 2π and let

$$\varphi_x(t) = \varphi(t) = f(x+t) + f(x-t) - 2s.$$

We denote by Δ a constant $\geqq 1$.

G. H. Hardy and J. E. Littlewood [3] proved the following theorem:

THEOREM A. *If $\varphi(t)$ satisfies*

$$\int_0^t |\varphi(u)|du = o\left(t/\log\frac{1}{t}\right) \tag{1.1}$$

and

$$\int_0^t |d\{u^\Delta\varphi(u)\}| = O(t), \quad 0 \leqq t \leqq \eta, \tag{1.2}$$

then the Fourier series of $f(t)$ converges to s at a point x.

Recently G. Sunouchi [5], [6], [7] proved the following theorems:

THEOREM B. *If $\varphi(t)$ satisfies the condition* (1.2) *and*

$$\int_0^t \varphi(u)du = o(t^\Delta), \tag{1.3}$$

then the Fourier series of $f(t)$ converges to s at a point x.

THEOREM C. *If satisfies the condition* (1.3) *and*

$$\lim_{k\to\infty}\limsup_{u\to 0}\int_{(ku)^{1/\Delta}}^{\eta} \frac{|\varphi(t+u)-\varphi(t)|}{t}dt = 0, \tag{1.4}$$

then the Fourier series of $f(t)$ converges to s at a point x.

THEOREM D. *If $\varphi(t)$ satisfies the conditions* (1.1) *and* (1.4), *then the Fourier series of $f(t)$ converges to s at a point x.*

THEOREM E. *Let us suppose that*

(1.5) *$\theta(x)$ is a positive differentiable function with $\theta'(x) > 0$,*

$$\Theta(x) = \int^x \frac{du}{u\theta(u)} \qquad \textit{increases with } x, \tag{1.6}$$

$$\mu(x, c) = 1/\Theta^{-1}(\Theta(x) - c). \tag{1.7}$$

Let $f(t)$ be an even integrable function with mean value zero and its