ON THE RIESZ SUMMABILITY OF FOURIER SERIES

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(Received February 5, 1956)

1. Let $\varphi(t)$ be an even integrable function with period 2π and let

$$\varphi(t) \sim \sum_{n=1}^{\infty} a_n \cos nt, \qquad a_0 = 0, \tag{1. 1}$$

$$\varphi_\alpha(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \varphi(u)(t-u)^{\alpha-1} du \qquad (\alpha > 0). \tag{1. 2}$$

The Fourier series (1. 1) is said to be summable $(\exp(\log \omega)^\lambda, \mu)$ to zero at $t = 0$ if

$$C_{\lambda,\mu}(\omega) = o\{\exp \mu(\log \omega)^\lambda\} \qquad \text{as } \omega \to \infty, \tag{1. 3}$$

where

$$C_{\lambda,\mu}(\omega) = \sum_{n<\omega} \{\exp(\log \omega)^\lambda - \exp(\log n)^\lambda\}^\mu a_n \qquad (\lambda, \mu > 0, \omega \geqq 2). \tag{1 .4}$$

Concerning this summability F.T. Wang [3], [4] has proved the following theorems:

Theorem A. *If*

$$\varphi_1(t) = o(t/\log(1/t)) \qquad as\ t \to 0, \tag{1. 5}$$

then the Fourier series (1, 1) *is summable* $(\exp(\log \omega)^2, 1 + \delta)$ *to zero at* $t = 0$ *for any* $\delta > 0$.

Theorem B. *If* $\alpha > 0$, *and*

$$\varphi_\alpha(t) = o(t^\alpha/\log(1/t)) \qquad as\ t \to 0, \tag{1. 6}$$

then the Fourier series (1. 1) *is summable* $(\exp(\log \omega)^{1+1/\alpha}, \alpha+1)$ *to zero at* $t=0$.

These theorems are the generalization of the theorem due to F.T.Wang [1].

The obeject of this paper is to generalize the above theorems.

Theorem 1. *If* $\alpha > 0$, $\beta > 0$ *and*

$$\varphi_\alpha(t) = o\left\{t^\alpha \Big/ \left(\log \frac{1}{t}\right)^{\alpha\beta}\right\} \qquad as\ t \to 0, \tag{1. 7}$$

then the Fourier series (1. 1) *is summable* $(\exp(\log \omega)^{1+\beta}, \alpha + \delta)$ *to zero at* $t = 0$, *where* $\alpha + \delta > [\alpha] + 1$ *for fractional* α.