ABSOLUTE SUMMABILITY OF RADEMACHER SERIES

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1. Introduction. A series $\sum a_n$ is said to be absolutely summable (A) or briefly summable $|A|$ if the series $\sum a_n x^n$ is convergent for $0 \leqq x < 1$ and its sum-function $\phi(x)$ satisfies that

$$\int_0^1 |\phi'(x)|\, dx < \infty. \tag{1.1}$$

This summability has been studied by many writers. Recently T. M. Flett introduced an extension of this summability for $k \geqq 1$ replacing the condition (1.1) by the condition

$$\int_0^1 (1-x)^{k-1} |\phi'(x)|^k\, dx < \infty, \tag{1.2}$$

which has an important significance as well as (1.1) in the theory of Fourier series (cf. [1] where references are given). T. M. Flett called this summability "summability $|A|_k$" where $k \geqq 1$. At the same time he gave an extension for summability $|C|$—— the absolute Cesàro summability——, that is, the series $\sum a_n$ is called summable $|C, \alpha|_k$, where $k \geqq 1$, $\alpha > -1$, if the series

$$\sum n^{k-1} |\sigma_{n-1}^{\alpha} - \sigma_n^{\alpha}|^k \tag{1.3}$$

is convergent, σ_n^{α} being the n th Cesàro mean of order α of the series $\sum a_n$. The summability $|C, \alpha|_1$ is the ordinary absolute summability $|C, \alpha|$.

Among the many theorems on the absolute summability, one of the most interesting is the so-called high-indices theorem. By the Zygmund high-indices theorem [5], if the series $\sum a_n$ is lacunary, that is, its terms are all zero except for the terms with indices $n_0 = 0 < n_1 < n_2 < \ldots.$ such that $n_{j+1}/n_j \geqq q > 1$ $(j = 1, 2, \ldots.)$, and if the series is summable $|A|$, then it turns out to be absolutely convergent or summable $|C, 0|$. Flett [2] studied an extension of this result to the summability $|A|_k$ and gave an inequality corresponding to that of the Zygmund theorem [5]. But he has left open the problem: If the series $\sum a_n$ is lacunary and summable $|A|_k$, then is it summable $|C, 1-1/k|_k$ where $k > 1$?

The main purpose of this paper is to give a negative answer to this problem. Therefore the Flett inequality ([2] Theorem 1) is the best possible one of this sense.