A NOTE ON THE LIE ALGEBRAS OF ALGEBRAIC GROUPS

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0. In his book [1] C. Chevalley defined the replicas for any elements of Lie algebras of algebraic groups of matrices which are defined over fields of characteristic 0, and he characterized algebraic subalgebras as those subalgebras of the general linear algebras which are closed with respect to "replica operation" i.e. those which contain all replicas of any elements of themselves. In this paper we shall define the replica in the case of any algebraic groups defined over fields of characterestic 0 and show that the same characterization of algebraic subalgebras is true in this case too.

1. Let G be a connected algebraic group[1]; let Ω (G) be the field of rational functions on G where Ω is the universal domain; let $k(G)$ be the subfield of $\Omega(G)$ consisting of all rational functions defined over k where k is a field of definition for G. Then $\Omega(G)$ is the union of $k(G)$ for all fields k of definition for G, and the mapping $f \to f(p)$ is a k-isomorphism of $k(G)$ onto $k(p)$ where p is a generic point over k on G. Suppose that G is given by $[V_\alpha, \mathfrak{F}_\alpha, T_{\beta\alpha}]$, and let $\xi_1, \dots, \xi_N$ be the coordinate functions relative to V_α, i.e. $\xi_i(p) = x_i$, where (x) is the representative of p in V_α. Then we have $\Omega(G) = \Omega(\xi)$ and $k(G) = k(\xi)$.

For any $p \in G$ we denote by $\mathfrak{o}_p$ the local ring of p on G. Let $\mathfrak{m}_p$ be the maximal ideal of $\mathfrak{o}_p$. By a tangent vector to G at p we mean an Ω-linear mapping X_p of $\mathfrak{o}_p$ into Ω such that for $f_1, f_2 \in \mathfrak{o}_p$ we have

$$X_p(f_1f_2) = (X_pf_1)\, f_2(p) + f_1(p)(X_pf_2).$$

If k is a field of definition for G such that p is rational over k, then X_p is said to be rational over k if X_p maps $\mathfrak{o}_p \cap k(G)$ into k. An Ω-derivation D is said to be finite at p if D maps $\mathfrak{o}_p$ into itself. In this case D induces a tangent vector D_p to G at p such that $D_p f = (Df)(p)$ for $f \in \mathfrak{o}_p$, which is called the local component of D at p. If further D is defined over k and maps $\mathfrak{m}_p \cap k(G)$ into itself, we have a k-derivation X of $k(p)$ such that $Xf(p) = (Df')(p)$ for $f(p) \in k(p)$, where f' is an element of $\mathfrak{o}_p \cap k(G)$ such that $f'(p) = f(p)$.

Let ρ be a rational mapping of G into another connected algebraic group G'. If ρ is generically surjective, we get an Ω-isomorphism ρ^* of $\Omega(G')$ into $\Omega(G)$ such that $\rho^*f(x) = f(\rho(x))$ for $f \in \Omega(G')$, where x is a generic point on G over some field of definition for G, G', ρ, and f. For $p \in G$ let R_p be the right translation, let L_p be the left translation, and let $\iota(p)$ be the inner

1) As for the terminology and preliminary results, cf. Nakano [2] and Rosenlicht [3], [4].