ON DIFFERENTIABLE MANIFOLDS WITH CERTAIN STRUCTURES WHICH ARE CLOSELY RELATED TO ALMOST CONTACT STRUCTURE I

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1. Introduction. Let M^{2n} be a differentiable manifold. If there exists a tensor field ϕ of type $(1,1)$ over M^{2n} such that

$$\phi^a_b\phi^b_c = -\delta^a_c, \qquad (a, b, c = 1, 2, \ldots\ldots, 2n)$$

then M^{2n} is said to be a differentiable manifold with almost complex structure. (Tensor fields of the form given above may exist only for some manifolds with even dimension.) We shall call ϕ the fundamental collineation of the almost complex structure. The set of differentiable manifolds with almost complex structure is wider than the set of complex manifolds.

Every differentiable manifold with almost complex structure ϕ admits a poistive definite Riemannian metric g such that

$$g_{ab}\phi^a_c\phi^b_d = g_{cd},$$

and the manifold is said to have Hermitian structure and to be a Hermitian manifold. Making use of the metric g and a skew symmetric tensor

$$\phi_{ab} = g_{ae}\phi^e_b,$$

we can reduce the structural group of the tangent bundle of any manifold with almost complex structure to the unitary group $U(n)$. The converse is also true.

Differentiable manifolds with almost complex structure or almost Hermitian structure were investigated by C. Ehresmann [1], B. Eckmann, A. Frölicher [2] and others and were interesting topics on differential geometry and topology in these fifteen years.

On the other hand, let M^{2n+1} be a $(2n+1)$-dimensional differentiable manifold. If there exists a tensor field ϕ^i_j, contravariant and covariant vector fields ξ^i and η_j over M^{2n+1} such that

$$(1.1) \qquad \xi^i\eta_i = 1, \qquad (i, j, k = 1, 2, \ldots\ldots, 2n+1)$$

$$(1.2) \qquad \operatorname{rank} |\phi^i_j| = 2n,$$

$$(1.3) \qquad \phi^i_j\xi^j = 0,$$