ON THE $(K, 1, \alpha)$ METHODS OF SUMMABILITY

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1. Zygmund [5] defined the $(K, 1)$ method of summability of a series. This method has the several similar properties to those of the Riemann's $(R, 1)$ method of summability. Concerning the method $(R, 1)$, we have defined, in the paper [1], the Riemann-Cesàro method $(R, 1, \alpha)$ which reduces the method $(R, 1)$ when $\alpha = -1$. In this note, by the analogous method, concerning the method $(K, 1)$, we shall define the new methods of summability and show that the new methods have the similar properties to those of the methods $(R, 1, \alpha)$.

Let α be a real number such that $-1 \leqq \alpha \leqq 0$, and let s_n^α be the Cesàro sum, of order α, of a series $\sum_{n=0}^{\infty} a_n$ with $a_0 = 0$. If the series in

$$\tau(\alpha, t) = t^{\alpha+1} \sum_{n=1}^{\infty} s_n^\alpha \int_t^\pi \frac{\sin nx}{2 \tan x/2} dx$$

converges in some interval $0 < t < t_0$, and if

$$\lim_{t \to 0+} \tau(\alpha, t) = B_\alpha s,$$

where

$$B_\alpha = \begin{cases} \pi/2 & \alpha = -1 \\ (\alpha + 1)^{-1} \sin(\alpha + 1)\pi/2 & -1 < \alpha < 0 \\ 1 & \alpha = 0, \end{cases}$$

then, we will say that the series $\sum_{n=0}^{\infty} a_n$ is evaluable $(K, 1, \alpha)$ to s. When $\alpha = -1$, the method $(K, 1, \alpha)$ reduces the method $(K, 1)$.

2. The above constant B_α is obtained if we consider the $(K, 1, \alpha)$ transform of the series

$$0 + 1 + 0 + 0 + \ldots\ldots .$$

For $\alpha = -1$, it is obvious that $B_\alpha = \pi/2$. For $-1 < \alpha < 0$, since, A_n^α denoting the Andersen notation,

$$\tau(\alpha, t) = t^{\alpha+1} \sum_{n=1}^{\infty} A_{n-1}^\alpha \int_t^\pi \frac{\sin nx}{2 \tan x/2} dx$$