# ON GROUPS OF AUTOMORPHISMS OF FINITE FACTORS

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**Introduction.** One of the important questions in the theory of the crossed products of rings of operators is the following: *Is the crossed product of a finite factor* $\mathbf{M}$ *also a finite factor for any group* $G$ *of automorphisms of* $\mathbf{M}$? The answer for this question is negative in general ([4]), and some kinds of conditions on $G$ under which the crossed product is a factor have been obtained ([4]). In § 2 we shall deal with this question when $G$ is abelian, and sharpen the results in [2]. In §3 we shall consider the behaviour of the action of $G$ in the crossed product and give a condition on $G$ under which the crossed product is a factor.

**1.** Throughout this paper, we assume that all $W^*$-algebras are finite factors with the invariants $C = 1$. An automorphism of a $W^*$-algebra means a $^*$-automorphism, and a group of outer automorphisms of a $W^*$-algebra is a group of automorhisms all member of which are outer automorphisms except the unit. The unit of a group will be denoted by $e$. $R(a_\lambda | \lambda \in \Lambda)$ means the $W^*$-algebra generated by the family of operators $a_\lambda$ $(\lambda \in \Lambda)$.

For convenience sake, we shall explain the construction of the crossed product. Let $\mathbf{M}$ be a finite factor with the invariant $C = 1$ on a Hilbert space $\mathbf{H}$ and $G$ a group of automorphisms of $\mathbf{M}$. Let $\varphi$ be a separating and generating trace vector for $\mathbf{M}$. For each $\sigma \in G$ we define

$$u_\sigma(a\varphi) = a^{\sigma^{-1}}\varphi \qquad \text{for all } a \in \mathbf{M}$$

where $a^\tau$ is the image of $a$ by an automorphism $\tau$. Then $u_\sigma$ can be extended to a unitary operator on $\mathbf{H}$ which will be also denoted by $u_\sigma$, and $\sigma \rightarrow u_\sigma$ is a faithful unitary representation of $G$ on $\mathbf{H}$ such that

$$u_\sigma^* a u_\sigma = a^\sigma \qquad \text{for all } a \in \mathbf{M}.$$

Now consider the Hilbert space $\mathbf{H} \otimes l_2(G)$. If we choose the complete orthonormal system $\{\varepsilon_\alpha\}_{\alpha\in G}$ in $l_2(G)$ such as

$$\varepsilon_\alpha(\gamma) = \begin{cases} 1 & \text{if } \quad \gamma = \alpha \\ 0 & \text{otherwise,} \end{cases}$$

each vector of $\mathbf{H}\otimes l_2(G)$ is expressed in the form $\sum \varphi_\alpha \otimes \varepsilon_\alpha$ where $\varphi_\alpha \in \mathbf{H}$