ON HARMONIC TENSORS IN AN ALMOST TACHIBANA SPACE

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1. Introduction. In a compact Kählerian space, a skew-symmetric pure tensor field of type $(0, q)$ is harmonic if and only if it is analytic, [5] [6][1]. But if the space is non-Kählerian, an (almost) analytic tensor is not necessarily harmonic.

For this problem, S.Tachibana [4] proved the following

THEOREM. *In a compact almost Tachibana space, a necessary and sufficient condition that a vector be covariant almost analytic is that* v_j *and* $\tilde{v}_j = \varphi_j{}^l v_l$ *are both harmonic.*

In this paper, we shall generalize this theorem to a tensor of type $(0, q)$

MAIN THEOREM. *In a compact almost Tachibana space, a necessary and sufficient condition that a skew-symmetric pure tensor* $T_{(j)}$ *of type* $(0, q)$ *be almost analytic is that* $T_{(j)}$ *and* $\tilde{T}_{(j)} \equiv \varphi_{j_s}{}^s T_{j_q \ldots s \ldots j_1}$ *are both harmonic.*

This main theorem follows from the following two lemmas.

LEMMA A[2]. *In an almost complex space, if skew-symmetric pure tensors* $T_{(j)}$ *and* $\tilde{T}_{(j)}$ *of type* $(0, q)$ *are both closed, then they are almost analytic.*

LEMMA B. *In a compact almost Tachibana space, if a skew-symmetric pure tensor* $T_{(j)}$ *of type* $(0, q)$ *is almost analytic, then it is harmonic.*

In §2 we shall give some well known lemmas concerning an almost analytic tensor in an almost complex space and prove Lemma A. In §3 we shall deal with an almost Tachibana space (which is called a K-space by some writers) and give two lemmas obtained by S.Sawaki [3]. In the last section, we shall prove Lemma B.

2. Almost analytic tensors. Let X_{2n} be a $2n$-dimensional real differentiable manifold of class C^∞, with local coordinate $\{x^i\}$, admitting an almost complex structure defined by the tensor field $\varphi_i{}^h$ of type (1. 1) satisfying

$$(2.1) \qquad \varphi_i{}^l \varphi_l{}^h = -\delta_i^h, \qquad h, i, \ldots = 1, 2, \ldots\ldots, 2n.$$

1) The numbers in brackets refer to References at the end of the paper.
2) This lemma the author owes to Dr. S.Tachibana. Cf., S.Tachibana [5], p.213.