ON EXCEPTIONAL VALUES OF ENTIRE AND MEROMORPHIC FUNCTIONS

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1. Let $F(z)$ be a meromorphic function and let $T(r, F)$ be its Nevanlinna characteristic function. Let $N(r, a) = N(r, F - a)$; $N(r, F) = N(r, \infty)$ have the usual meaning in the Nevanlinna theory.

Define

$$\delta(a) = 1 - \limsup_{r\to\infty} \frac{N(r, a)}{T(r, F)},$$

$$\Delta(a) = 1 - \liminf_{r\to\infty} \frac{N(r, a)}{T(r, F)}.$$

If $\delta(a) > 0$ we say that a is an exceptional value for $F(z)$ in the sense of Nevanlinna (e. v. N); and if $\Delta(a) > 0$ we call a as an e. v. V (exceptional value in the sense of Valiron).

2. Let $f(z)$ be an entire function and let

$$\mu(r, f) = \mu(r) = \operatorname*{Min}_{|z| = r} | f(z) |.$$

It is clear that if 0 is an asymptotic value for $f(z)$ then $\mu(r) \to 0$ as $r \to \infty$. We show that the converse is not true. We prove:

THEOREM 1. *For an entire function $f(z)$, the minimum modulus $\mu(r)$ tending to zero does not imply that 0 is an asymptotic value.*

LEMMA *If 0 is an e. v. N for the entire function $f(z)$ then $\mu(r) \to 0$ as $r \to \infty$.*

PROOF. In the terminology of Nevanlinna

$$m\left(r, \frac{1}{f}\right) = m(r, 0) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ \left| \frac{1}{f(re^{i\theta})} \right| d\theta.$$

Hence
$$m(r, 0) \leqq \log^+ \frac{1}{\mu(r)}.$$

But
$$\liminf_{r\to\infty} \frac{m(r, 0)}{T(r, f)} > 0,$$