ON THE ALMOST-COMPLEX STRUCTURE OF TANGENT BUNDLES OF RIEMANNIAN SPACES

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Recently we can see several papers concerning almost-Kählerian spaces, but it seems for the authors that there does not exist a non-Kählerian global example of such a space. In this paper we shall show that the tangent bundle space $T(M^n)$ of any non-flat Riemannian space M^n always admits an almost-Kählerian structure which is not Kählerian. This is done by making use of the almost-complex structure of $T(M^n)$ owing to T. Nagano [1][1)] and of the Riemannian metric of $T(M^n)$ owing to S.Sasaki [2]. By virtue of this structure we shall also see that an infinitesimal affine transformation has an almost-analytic property in a certain sense.

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1. Almost-Kählerian spaces. Let us consider a $2n$-dimensional differentiable manifold admitting a tensor field $\varphi_\kappa^{\ \lambda}$ such that $\varphi_\lambda^{\ \alpha}\varphi_\alpha^{\ \kappa} = -\delta_\lambda^\kappa$ [2)]. Such a manifold is called an almost-complex space and it is said that the tensor field assigns to the manifold an almost-complex structure. An almost-complex structure is called to be integrable if the tensor field defined by

$$N_{\mu\lambda}^{\ \ \kappa} = \varphi_\mu^{\ \alpha}(\partial_\alpha\varphi_\lambda^{\ \kappa} - \partial_\lambda\varphi_\alpha^{\ \kappa}) - \varphi_\lambda^{\ \alpha}(\partial_\alpha\varphi_\mu^{\ \kappa} - \partial_\mu\varphi_\alpha^{\ \kappa})$$

vanishes identically.

An infinitesimal transformation V^κ of an almost-complex space is called to be almost-analytic [3] if it satisfies $\underset{V}{\pounds}\, \varphi_\lambda^{\ \kappa} = 0$, where $\underset{V}{\pounds}$ means the operator of Lie derivation.

An almost-complex space always admits a Riemannian metric $G_{\mu\lambda}$ such that

$$G_{\beta\alpha}\varphi_\mu^{\ \beta}\varphi_\lambda^{\ \alpha} = G_{\mu\lambda} \tag{1.1}$$

which is equivalent to the fact that $\varphi_{\mu\lambda}$ defined by $\varphi_{\mu\lambda} = \varphi_\mu^{\ \alpha}G_{\lambda\alpha}$ is skew-symmetric or that $G_{\mu\lambda}$ is hybrid [3].

An almost-complex space with such a Riemannian metric is called an almost-Hermitian space and the differential form $\varphi = (1/2)\varphi_{\mu\lambda}dx^\mu \wedge dx^\lambda$ is called the fundamental form. If the form is closed, the almost-Hermitian

1) The number in brackets refers to Bibliography at the end of the paper.
2) $\lambda, \mu, \nu, \alpha, \cdots\cdots = 1, 2, \cdots\cdots, 2n.$