SOME REMARKS ON SPACE WITH A CERTAIN CONTACT STRUCTURE

MASAFUMI OKUMURA

(Received October, 1, 1961)

Introduction. Recently S. Sasaki [3][1] defined the notion of (ϕ, ξ, η, g) structure of a differentiable manifold. Further, S. Sasaki and Y. Hatakeyama [4] [5] showed that the structure is closely related to contact structure. By means of this notion, it is shown that a space with a contact structure can be dealt with as we deal with an almost complex space. So, by similar manner, some problems discussed in the latter space may be considered in the former. On the other hand, S. Tachibana [6] [7] proved many interesting theorems in an almost complex space. In this paper, the present author tries to study, in the space with a certain contact structure, the problem corresponding to S. Tachibana's results. We shall devote § 1 to preliminaries and in this section introduce a normal contact structure. In § 2, we ennumerate identities which will be useful in the later sections. We shall prove in § 3 that a space with a normal contact structure satisfying $\nabla_k R_{ji} = 0$ be necessarily an Einstein one and that a symmetric space with a normal contact structure reduces to the space of constant curvature respectively. The differential form $\widehat{R}$ is dealt with in § 4, and in this section, we shall show a necessary and sufficient condition that the space be an Einstein space by means of the form $\widehat{R}$. Finally in § 5, we introduce a certain type of (ϕ, η, g)-connection with respect to which the fundamental tensors ϕ_{ji}, η_j and g_{ji} are all covariant constant.

The present author wishes to express his hearty thanks to Prof. S. Tachibana for his many valuable advices and several discussions.

1. Preliminaries. Let M be an n-dimensional real differentiable manifold. If there exist a tensor field $\phi_j{}^i$, contravariant and covariant vector fields ξ^i, η_j over M such that

$$\xi^i \eta_i = 1, \tag{1. 1}$$

$$\text{rank } |\phi_j{}^i| = n - 1, \tag{1. 2}$$

$$\phi_j{}^i \xi^j = 0, \tag{1. 3}$$

$$\phi_j{}^i \eta_i = 0, \tag{1. 4}$$

$$\phi_j{}^i \phi_k{}^j = - \delta_k^i + \xi^i \eta_k, \tag{1. 5}$$

1) Numbers in brackets refer to the references at the end of the paper.