INDECOMPOSABILITY OF DIFFERENTIABLE MANIFOLDS

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1. Let X_n and X_m be compact orientable differentiable manifolds of dimension n and m respectively. Then their product $X_{n+m} = X_n \times X_m$ is also a compact orientable differentiable manifold. Conversely, if a X_{n+m} admits such a decomposition we say that X_{n+m} is decomposable and if not, we say that X_{n+m} is indecomposable. In this paper we shall study the conditions on which a compact orientable differentiable manifold X_{4k} be indecomposable. According to the cobordism theory such a manifold admits the following cobordism decomposition :

$$(1.\ 1) \qquad X_{4k} \sim \sum_{i_1+\cdots+i_t=k} A^k_{i_1\cdots i_t} P_{2i_1}(c)\cdots P_{2i_t}(c) \qquad \text{mod torsion}$$

where $P_j(c)$ denotes the complex projective space of complex dimension j. In general A's are rational numbers and in particular $A^1_1, A^2_2, A^2_{11}, A^3_3, A^3_{21}, A^3_{111}, 3A^4_4, A^4_{31}, A^4_{22}, A^4_{211}, 3A^4_{1111}$ are integers ([4], [7]). There is no torsion in the cases $k = 1, 2, 3$ and in general the torsions are of order 2 ([5]). We shall use the following multiplicative series:

$$(1.\ 2) \qquad \sum_i \Gamma_i(y, p_1, \cdots, p_i) = \prod_j \frac{\sqrt{r_j}}{\mathrm{tgh}\sqrt{r_j}}(1 + y\ \mathrm{tgh}^2\sqrt{r_j})$$

where

$$(1.\ 3) \qquad p = \sum_i p_i = \prod_j (1 + r_j)$$

and p_i denotes the Pontrjagin class of dimension $4i$.

First of all we have the following

THEOREM 1. *If $p_k[X_{4k}] \neq 0$ and the polynomial with regard to y*

$$(1.\ 4) \qquad \Gamma_k(y, p_1, \cdots, p_k)\ [X_{4k}]$$

is irreducible in the rational field, then such an X_{4k} is indecomposable.

PROOF. The polynomial $\Gamma_k(y, p_1, \cdots, p_k)\ [X_{4k}]$ is of integral coefficients and its order does not exceed k ([2]). It is clear that the coefficient of y^k is equal to $p_k[X_{4k}]$. Suppose that X_{4k} decomposes as follows:

$$(1.\ 5) \qquad X_{4k} = X_{4s} \times X_{4t}.$$