$(\mathfrak{R}, p, \alpha)$ METHODS OF SUMMABILITY

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1. G. Sunouchi [3] has recently introduced some new methods of summability which are regular. These are defined in the following way. A series $\sum_{n=0}^{\infty} a_n$ is said to be summable $(\mathfrak{R}, \alpha)$ to s if the series in

$$f_1(t) = a_0 + \left(\int_0^\infty \frac{\sin x}{x^{\alpha+1}}\, dx\right)^{-1} \sum_{n=1}^{\infty} a_n \int_t^\infty \frac{\sin nu}{n^\alpha u^{\alpha+1}}\, du, \quad 0 < \alpha < 1,$$

converges in some interval $0 < t < t_0$ and $f_1(t) \to s$ as $t \to 0+$. A series $\sum_{n=0}^{\infty} a_n$ is said to be summable $(\mathfrak{R}^*, \alpha)$ to s if the series in

$$f_2(t) = a_0 + \left(\int_0^\infty \frac{\sin^2 x}{x^{\alpha+1}}\, dx\right)^{-1} \sum_{n=1}^{\infty} a_n \int_t^\infty \frac{\sin^2 nu}{n^\alpha u^{\alpha+1}}\, du, \quad 0 < \alpha < 1,$$

converges in some interval $0 < t < t_0$ and $f_2(t) \to s$ as $t \to 0+$.

It is purpose of this paper to obtain information about these Sunouchi's methods of summability and generalization of them. Throughout this paper, p denotes a positive integer and α denotes a real number, not necessarily an integer, such that $0 < \alpha < p$. Let us put

$$C_{p,\alpha} = \int_0^\infty \frac{\sin^p x}{x^{\alpha+1}}\, dx,$$

$$\varphi(n, t) \equiv \varphi(nt) \equiv (C_{p,\alpha})^{-1} \int_{nt}^\infty \frac{\sin^p x}{x^{\alpha+1}}\, dx = (C_{p,\alpha})^{-1} \int_t^\infty \frac{\sin^p nu}{n^\alpha u^{\alpha+1}}\, du.$$

Then a series $\sum_{n=0}^{\infty} a_n$ will be said to be summable $(\mathfrak{R}, p, \alpha)$ to s if the series in

$$f(p, \alpha, t) = a_0 + \sum_{n=1}^{\infty} a_n \varphi(nt)$$

converges in some interval $0 < t < t_0$ and $f(p, \alpha, t) \to s$ as $t \to 0+$. Under this definition, the $(\mathfrak{R}, \alpha)$ method and the $(\mathfrak{R}^*, \alpha)$ method are reduced to the

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