SATURATION IN THE LOCAL APPROXIMATION

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§1. **Introduction**. Let $f(x)$ be integrable in $(-\pi, \pi)$ and periodic with period 2π, and let its Fourier series be

$$S(f) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{1}$$
$$= \sum_{n=0}^{\infty} A_n(x).$$

The Poisson integral of $f(x)$

$$f(r, x) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x+t)\, P_r(t)\, dt \tag{2}$$
$$= \sum_{n=0}^{\infty} A_n(x)\, r^n$$

satisfies the Laplace equation

$$\left(r \frac{\partial}{\partial r}\right)^2 f(r, x) + \frac{\partial^2}{\partial x^2} f(r, x) = 0$$

inside the unit circle. Moreover if $f(x)$ is continuous over (a, b), then $f(r, x)$ tends uniformly to $f(x)$ as $r \to 1$ over (a', b') situated inside (a, b). What can we say about $f(x)$, when the rapidity of approximation of $f(x)$ by $f(r, x)$ is given ? If (a, b) is $(-\pi, \pi)$ and

$$f(r, x) - f(x) = o(1 - r),$$

uniformly if and only if $f(x)$ is a constant and

$$f(r, x) - f(x) = O(1 - r)$$

uniformly if and only if $\widetilde{f}(x)$ satisfies the Lipschitz condition, see G. Sunouchi and C. Watari [5]. This is a saturation theorem.

We shall investigate this problem over an interval (a, b) situated inside $(-\pi, \pi)$ in §2 of this note. In §3 and §4, we consider such problem about