THE TENSOR PRODUCT OF FUNCTION ALGEBRAS

NOZOMU MOCHIZUKI

(Received January 27, 1965)

1. Introduction. We consider the tensor product of two function algebras A and B on compact Hausdorff spaces X and Y, respectively, where by function algebras we shall mean uniformly closed subalgebras of the continuous complex-valued functions which contain the constants and separate the points.

Let $A \odot B$ be the algebraic tensor product of A, B and let $\sum_{i=1}^{n} f_i \otimes g_i \in A \odot B$, then, by $\left(\sum_{i=1}^{n} f_i \otimes g_i\right)(x,y) = \sum_{i=1}^{n} f_i(x)\, g_i(y)$ for $(x,y) \in X \times Y$, $\sum_{i=1}^{n} f_i \otimes g_i$ belongs to $C(X\times Y)$. Let $A \hat{\otimes} B$ be the completion of $A \odot B$ under the λ-norm.*) The λ-norm is identical with the usual uniform norm on $X \times Y$ and $C(X) \hat{\otimes} C(Y) = C(X \times Y)$. Thus, $A \hat{\otimes} B$ is a Banach algebra. Further, it is easily seen that $A \hat{\otimes} B$ becomes a function algebra on $X \times Y$, which we shall denote by $\mathfrak{A}$. Now, it will be natural to ask what properties of A and B are inherited to $\mathfrak{A}$, or conversely. We shall show that the Šilov boundary and the Choquet boundary of $\mathfrak{A}$ are represented exactly as Cartesian products of such subsets. Each Gleason part of the maximal ideal space of $\mathfrak{A}$ is also a Cartesian product of parts of maximal ideal spaces of A, B respectively. Even if both A and B are dirichlet algebras ($A \neq C(X)$, $B \neq C(Y)$), $\mathfrak{A}$ is far from being dirichlet. However, the maximal ideal space of $\mathfrak{A}$ remains to have an analytic structure, where analytic functions of two complex variables on the open unit bicylinder are involved.

The author is indebted to Prof. T. Turumaru and Mr. J. Tomiyama for valuable conversations on the subject of this paper.

2. Tensor product $A \hat{\otimes} B$. In what follows, we denote by ∂_A, $\mathfrak{M}(A)$ and $M(A)$ the Šilov boundary, the maximal ideal space and the Choquet boundary of A, respectively. The closed unit disk $\{z : |z| \leqq 1\}$ of the complex plane is denoted by D and its interior $\{z : |z| < 1\}$ by D^i. We use the symbol T for the unit circle.

*) Let A^*, B^* be the conjugate spaces of A, B respectively. For $\sum f_i \otimes g_i \in A \odot B$, the λ-norm is defined by $\|\sum f_i \otimes g_i\|_\lambda = \sup|\sum \varphi(f_i)\psi(g_i)|$ where φ, ψ run over the unit balls of A^*, B^* respectively ([13]).