A THEOREM ON REGULAR VECTOR FIELDS AND ITS APPLICATIONS TO ALMOST CONTACT STRUCTURES

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(Received May 6, 1965)

Introduction. In the paper [1], Boothby-Wang dealt with the period function λ of the associated vector field of the regular contact form on a compact contact manifold and proved that λ is differentiable and constant ([6]).

In this note we prove a theorem on the proper and regular vector field, by this we can give a simple proof to one of their result. Moreover, as a natural consequence, this procedure enables us to generalize Morimoto's theorem (Theorem 5, [4]), concerning the period function on a normal almost contact manifold.

Auther wishes to express his best thanks to Professor S. Sasaki who kindly gave him the guidance to study this problem.

1. **Regular vector fields**. Let M be a connected differentiable manifold and X be a differentiable vector field on M such that X does not vanish everywhere. We assume that the distribution defined by X is regular and X is proper, i.e., X generates the global 1-parameter group $\exp tX(-\infty<t<\infty)$ of transformations of M. For the terminologies we refer to [5]. We can find always a 1-form w satisfying $w(X)=1$. Now the next assumption is that there exists a 1-form w such that $w(X)=1$ and $L(X)w=i(X)\,dw=0$, where $L(X)$ or $i(X)$ is the operator of the Lie derivative or interior product operator by X.

First we see that the quotient space M/X is a Hausdorff space, because X is proper and regular. Hence by Palais' theorem ([5], Chap. I, § 5), M/X is a differentiable manifold and the projection $\pi: M\rightarrow M/X$ is a differentiable map.

Let h be an arbitrary Riemannian metric in M/X. The tensor g in M defined by $g=\pi^{*}h+w\otimes w$ is easily seen to be a Riemannian metric in M, π^{*} and $\otimes$ denoting the dual of π and tensor product respectively. Clearly we have $g(X,X)=1$, and we see that the relation $L(X)g=0$ holds good. Namely X is a unit and Killing vector field with respect to g. Thus each trajectory of X is a geodesic and the parameter t in $\exp tX$ is nothing but the arc length of it.

Suppose that there is a point p and a positive number λ such that