Tôhoku Math. Journ.
Vol. 18, No. 2, 1966

A NOTE ON THE LITTLEWOOD-PALEY FUNCTION $g^*(f)$

SATORU IGARI

(Received February 28, 1966)

In a previous paper [1], we studied the functions of Littlewood-Paley, Lusin and Marcinkiewicz. And now we wish to consider a proof for the function g^* independently of the decomposition theorem on Fourier series. The function $g^*(x,f)$ is defined for f of $L(0,1)$ as follows;

$$g^*(x,f) = g^*(f) = \left(|\hat{f}_0|^2 + \sum_{n=1}^{\infty} \frac{|s_n(x,f)-\sigma_n(x,f)|^2}{n}\right)^{1/2},$$

where $\hat{f}_n$ is the n-th Fourier coefficient of f and s_n, σ_n denote the n-th partial sum of the Fourier series of f and its n-th Cesàro mean respectively.

THEOREM. *Let $1 < p < \infty$, then*

$$A_p\| f \|_p \leqq \| g^*(f)\|_p \leqq A_p'\| f \|_p$$

for all f of $L^p(0,1)$, A_p, A_p' being positive constants independent of f.

Let F_n be n-th Fejér kernel, that is,

$$F_n(x) = \sum_{\nu=-n}^{n}\left(1-\frac{|\nu|}{n+1}\right)e^{2\nu\pi ix} = \frac{1}{n+1}\left\{\frac{\sin(2n+1)\pi x}{\sin \pi x}\right\}^2$$

and let us denote $k_0(x)=1$ and

$$k_n(x) = \frac{F_{2n}(x)-F_n(x)}{c_n\sqrt{n}} \quad n = 1, 2, \cdots,$$

c_n being constants bounded away both from 0 and from infinity which will be defined later. Our proof composes, as that of [1] on the decomposition theorem, of estimation of the vector valued kernel $K(x) = (k_0(x), k_1(x), \cdots)$. Let us put $\mathfrak{K}f=K*f$ for f of $L(0,1)$ and $\mathfrak{L}g=\int <g(y), K(x-y)> dy$ for g of