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CLOSED GEODESICS ON CERTAIN RIEMANNIAN MANIFOLDS OF POSITIVE CURVATURE

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1. Introduction. To investigate the relations among geodesics, curvature and manifold structures is very interesting and important. The following theorem is well known (cf. Klingenberg [5]).

THEOREM. *Let M be a 2-dimensional complete simply connected Riemannian manifold with Gaussian curvature K, $0 < k \leqq K \leqq 1$, where k is a constant. Let G be a simple closed geodesic on M and $L(G)$ be its length. Then the following inequalities are satisfied:*

$$2\pi \leqq L(G) \leqq 2\pi/\sqrt{k}\ .$$

In particular, if there exists a closed geodesic of length 2π on M, then M is isometric to the sphere with constant curvature 1.

And if there exists a simple closed geodesic of length $2\pi/\sqrt{k}$ on M, then M is isometric to the sphere with constant curvature k.

In this paper we shall prove similar results in the higher dimensional case. By a geodesic triangle, we always mean a geodesic triangle composed of three shortest geodesic arcs.

THEOREM A. *Let M be an n-dimensional complete simply connected Riemannian manifold with sectional curvature K, $0 < k \leqq K \leqq 1$, where k is a constant. Let G be a closed geodesic which can be decomposed into a geodesic triangle and $L(G)$ be its length.*

Then the following inequalities are satisfied

$$2\pi \leqq L(G) \leqq 2\pi/\sqrt{k}\ .$$

In particular, if there exists a closed geodesic of length $2\pi/\sqrt{k}$ on M which can be decomposed into a geodesic triangle, then M is isometric to the