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COMPLEX-VALUED DIFFERENTIAL FORMS ON NORMAL CONTACT RIEMANNIAN MANIFOLDS

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Introduction. Almost contact manifolds have, as is well known, an aspect of the odd-dimensional version of almost complex manifolds, and especially normal contact Riemannian manifolds are looked upon as what correspond to Kähler manifolds. The purpose of this paper is to develop a theory on a normal contact Riemannian manifold parallel to that of Kähler manifold through the researches of complex-valued differential forms on the former.

After introducing several operators in the beginning section, in §2 we shall see that a trigrade structure, corresponding to the bigrade one in almost complex manifold, is naturally induced in the algebra of complex-valued forms on a contact Riemannian manifold. In §3 normal contact Riemannian manifolds are discussed from our standpoint of view and §4 is devoted to the investigations of harmonic forms on a compact normal contact Riemannian manifolds. The main result in this section is Theorem 4.4 which asserts the evenness of the r-th Betti numbers of the manifold for certain values of r. Some further researches are pursued in the last section.

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1. Preliminaries. Given an m-dimensional differentiable manifold M, we denote by $V(M)$ the space of complex-valued vector fields on M, by $A(M)$ that of complex-valued forms on M and by Π_r $(r=0,1,\cdots,m)$ the projection of $A(M)$ onto the subspace $A_r(M)$ of r-forms.

Let M be a contact Riemannian manifold with the structure (η, g). We denote the associated vector field by ξ and the $(1,1)$-tensor field by ϕ as usual. These are related in the following manner:

$$(1.1)\qquad \begin{cases} g(\xi, X) = \eta(X), \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0\,, \\ 2g(X, \phi Y) = d\eta(X, Y), \quad \phi^2 X = -X + \eta(X)\xi\,, \end{cases}$$