Tôhoku Math. Journ.
20(1968), 106-110.

ON THE TOPOLOGY OF COMPACT CONTACT MANIFOLDS

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(Received June 13, 1967, revised October 1, 1967)

1. Introduction. In an earlier paper [3] the author proved that the second betti number of a compact normal regular contact manifold of strictly positive curvature vanishes. This was subsequently strengthened by E.M.Moskal [5] by removing the regularity condition and another proof[2] was recently given by S.Tanno [9].

PROPOSITION 1. *The second betti number of a compact normal contact manifold of positive curvature is zero.*

In this paper we prove

THEOREM 1. *The second betti number of a compact normal regular contact manifold with non-negative sectional curvature is zero.*

The proof is based upon Proposition 2 below as well as the technique used to obtain Theorem 2 of [3].

Proposition 2 has other interesting consequences. Indeed an application of a result due to B.Kostant [4] yields

THEOREM 2. *A compact simply connected*[3] *(normal) contact symmetric space is isometric with a sphere.*

This also follows from a statement due to M.Okumura ([6], Theorem 3.2).

Employing Theorem 1 and Proposition 3 below, we obtain

THEOREM 3. *A compact torsion free 5-dimensional normal regular contact manifold with non-negative sectional curvature is homeomorphic with a sphere.*

1) Research supported by National Science Foundation Grant GP-5477.
2) An error in the proof originally due to Moskal led to the proof given by Tanno which is substantially the same.
3) D. Blair and the auther have shown that the fundamental group of a compact symmetric normal contact manifold is finite.