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A STRUCTURE THEOREM OF AUTOMORPHISMS OF VON NEUMANN ALGEBRAS

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In a recent few years automorphisms and derivations of C^*-algebras, especially of von Neumann algebras, have been investigated by various authors [1], [6], [7], [8], [11] and etc. We also are concerned with them in this paper. Our main purpose is to establish a structure theorem of (not necessarily $*$-preserving) automorphisms of von Neumann algebras, which we may call *the polar decomposition theorem.* It asserts that any automorphism of a von Neumann algebra is composed of an *inner* automorphism defined by an invertible positive operator and a $*$-automorphism in the unique way. This fact seems to assure us that the study of automorphisms may be reduced, in a sense, to that of $*$-preserving ones. For example, it can be said that the property that an automorphism is outer is due to its $*$-preserving part. Also we know, as its immediate consequence, that any automorphism of a von Neumann algebra is σ-strongly (that is, in the strongest operator-topology), as well as σ-weakly, bicontinuous. These arguments can be applied to those of isomorphisms between von Neumann algebras. Particularly we can give an answer to a problem proposed by S. Sakai in his lecture note [10], deciding that any isomorphism between von Neumann algebras is σ-strongly bicontinuous.

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1. Preliminaries.

It is assumed that all C^*-algebras under consideration have identities. Automorphisms, isomorphisms and representations of C^*-algebras are said to be $*$-automorphisms, $*$-isomorphisms and $*$-representations, respectively, if they are $*$-preserving.

Let H be a Hilbert space and $B(H)$ the C^*-algebra of all bounded linear operators on H. If an operator $a \in B(H)$ is invertible, $\rho_a(x)=axa^{-1}$ for $x \in B(H)$ is clearly an automorphism of $B(H)$. When A is a C^*-algebra acting on H, an automorphism ρ of A is said to be spatial if there is an invertible operator $a \in B(H)$ such that $\rho=\rho_a|A$, the restiiction of ρ_a on A. A spatial automorphism ρ of A is said to be inner if the above a can be chosen in A and to be outer if it is not inner. An automorphism ρ of A is said to be weakly inner (π-inner in [8]) if for any faithful $*$-representation π of A there