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ON THE ENESTRÖM-KAKEYA THEOREM

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The following result is well known in the theory of the distribution of zeros of polynomials.

THEOREM A. (Eneström-Kakeya). *If* $p(z)=\sum_{k=0}^{n} a_k z^k$ *is a polynomial of degree n such that*

$$a_n \geqq a_{n-1} \geqq a_{n-2} \geqq \cdots \geqq a_1 \geqq a_0 > 0, \tag{1}$$

then $p(z)$ *does not vanish in* $|z|>1$.

We may apply this result to $p(z/a)$ to obtain the following more general

THEOREM B. *If* $p(z)=\sum_{k=0}^{n} a_k z^k$ *is a polynomial of degree n such that for some* $a>0$

$$a_n \geqq a a_{n-1} \geqq a^2 a_{n-2} \geqq \cdots \geqq a^{n-1} a_1 \geqq a^n a_0 > 0\,, \tag{2}$$

then $p(z)$ *does not vanish in* $|z|>1/a$.

This is a very elegant result but it is equally limited in scope. The hypothesis is very restrictive and does not seem useful for applications. Our aim is to relax the hypothesis in several ways. In the literature there already exist ([1], [2, Theorem 3], [3]) some extensions of the Eneström-Kakeya theorem. In connection with Theorem A or the more general Theorem B the following questions appear to be very natural to ask.

Q.1. Can we drop the restriction that the coefficients are all positive and instead assume (2) to hold for the moduli of the coefficients ?