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INVARIANT SUBSPACES OF SOME NON-SELFADJOINT OPERATORS

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1. In [3] J. Schwartz has proved the following result.

Let T be an operator on a Hilbert space $\mathfrak{H}$ of more than one dimension. Write $T = A+iB$, where A and B are selfadjoint, and suppose that the imaginary part B of T belongs to one of the classes C_p, where $1 \leqq p < \infty$. Then the Hilbert space admits a proper closed subspace which is invariant under T.

The purpose of this note is to show that the above theorem may be generalized to an operator such as T is the sum of a normal operator A with some spectrum condition and a compact operator B with some condition. In fact, we shall show the following theorem:

THEOREM. *Let T be an operator on a Hilbert space $\mathfrak{H}$ of more than one dimension. Write $T=A+B$, where T is the sum of a normal operator A, whose spectrum lies on a Jordan curve J, which consists of a finite number of rectifiable smooth arcs, (it may well be the case that the spectrum separates the plane), and a compact operator B, which belongs to one of the classes C_p, where $1 \leqq p < \infty$. Then the Hilbert space admits a proper closed subspace which is invariant under T.*

Throughout the present note, an operator means a bounded linear operator on a Hilbert space $\mathfrak{H}$ which we assume to be separable. We denote by $\sigma(T)$, $\sigma_p(T)$, $\sigma_c(T)$, $\sigma_r(T)$ and $\rho(T)$ the spectrum, the point spectrum, the continuous spectrum, the residual spectrum and the resolvent set of an operator T respectively. For the sake of convenience, we shall list some results on the classes C_p ([2], Part II, Section XI. 9).

Let T be a compact operator on a Hilbert space and $H = (T^*T)^{1/2}$. Let $\mu_1, \mu_2, \cdots, \mu_n, \cdots$ be the eigenvalues of H, arranged in decreasing order and repeated according to multiplicity. We write $\mu_n(T)$ for the n-th eigenvalue