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EXCISION THEOREMS ON THE PAIR OF MAPS

HIDEO ANDO

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Introduction. Let $E_f \longrightarrow X \xrightarrow{f} Y$ be an extended fibration. Then the 1-1 and onto correspondence ε_f^{-1}: $\pi_1(V,f) \to \pi(V, E_f)$ is defined easily (section 2). Moreover, let Ψ be the pair-map (g_1, g_2): $f_1 \to f_2$ in (1.3); then the 1-1 and onto correspondence ε_Ψ^{-1}: $\pi_2(V, \Psi) \to \pi_1(V, f_{1,2})$ is defined (section 3). The object of this paper is to establish excision theorems on the pair of maps by applying ε_f^{-1} and ε_Ψ^{-1}. These excision theorems are described in section 5.

1. Preliminaries. Throughout this paper we consider the category of spaces of the homotopy type of CW-complexes with base points denoted by $*$, and all maps and homotopies are assumed to preserve base points.

PX is the space of paths in X emanating from $*$, and ΩX is the loop space. If f: $X \to Y$ is any map, $Y \cup_f CX$ is the space obtained by attaching to Y the reduced cone over X by means of f. X is embedded in CX by $x \to (x, 1)$, and ΣX is the reduced suspension. $X \times Y$ is the Cartesian product and $X \vee Y = X \times * \cup * \times Y$. Then the smash product $X \# Y$ is the quotient space $X \times Y / X \vee Y$.

By applying the mapping track functor, any map f: $X \to Y$ is converted into a homotopy equivalent fibre map p: $E \to Y$,

(1.1)
$$\begin{array}{ccccc} E_f & \xrightarrow{j_f} & X & \xrightarrow{f} & Y \\ \| & \simeq & \downarrow h & \text{comm.} & \| \\ E_f & \xrightarrow{i} & E & \xrightarrow{p} & Y, \end{array}$$

where $E = \{(x, \eta) \in X \times Y^I | f(x) = \eta(1)\}$, $p(x, \eta) = \eta(0)$,
$E_f = \{(x, \eta) \in X \times PY | f(x) = \eta(1)\}$, i = the inclusion map,
$j_f(x, \eta) = x$, $h(x) = (x, \eta_x)$ and $\eta_x(t) = f(x)$ for $t \in I$,
$\simeq$ in the left diagram means homotopy commutativity.